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**QCD Phenomenology in Tachyon Condensation
Theory**

Adamu Issifu

JOÃO PESSOA

- July 2021 -

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**QCD Phenomenology in Tachyon Condensation
Theory**

Adamu Issifu

This Thesis is Presented to the Department of
Physics (Postgraduate division), Federal Univer-
sity of Paraíba as part of the requirements to
acquire a Doctor of Philosophy Degree in Physics.

Area of Concentration: Quantum Field Theory
and Particle Physics.

Advisor: Prof. Dr. Francisco de Assis de Brito.

JOÃO PESSOA

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Ata da Sessão Pública da Defesa de Tese de
Doutorado do aluno **Adamu Issifu**, candidato
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Aos trinta dias do mês de julho do ano de dois mil e vinte e um, às 14:00, reuniram-se, remotamente, os membros da Banca Examinadora constituída para examinar o candidato ao grau de Doutor em Física na área de Física das Partículas Elementares e Campos, **Adamu Issifu**. A comissão examinadora foi composta pelos professores doutores: *Francisco de Assis de Brito* (UFPB), orientador e presidente da banca examinadora, *Dionísio Bazeia Filho* (UFPB), *Albert Petrov* (UFPB), *Luciano Melo Abreu* (UFBA), *Carlos Alberto Santos de Almeida* (UFC). Dando início aos trabalhos, o Prof. Francisco de Assis de Brito comunicou aos presentes a finalidade da reunião. A seguir, passou a palavra para que o candidato fizesse, oralmente, a exposição do trabalho de tese intitulado "*QCD Phenomenology in Tachyon Condensation Theory*". Concluída a exposição, o candidato foi arguido pela Banca Examinadora, que emitiu o seguinte parecer: "**aprovado**". Assim sendo, deve a Universidade Federal da Paraíba expedir o respectivo diploma de Doutor em Física na forma da lei. E para constar, eu, Jose Sérgio Trindade Silva, redigi esta ata que vai assinada por mim e pelos membros da Banca Examinadora. João Pessoa, Paraíba, **30 de julho de 2021**.

Prof. Dr. Francisco de Assis de Brito
Orientador – PPGF/UFPB

Prof. Dr. Dionísio Bazeia Filho
PPGF/UFPB

Prof. Dr. Albert Petrov
PPGF/UFPB

Prof. Dr. Luciano Melo Abreu
UFBA

Prof. Dr. Carlos Alberto Santos de Almeida
UFC

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Abstract

This thesis is intended to contribute to the long standing problems associated with theoretical QCD theory. We aim at touching on all possible QCD features such as; color confinement, asymptotic freedom and chiral symmetry breaking. We focus on QCD phenomenology by considering an effective theory for Dp -branes at tachyonic vacuum. We will highlight the behaviour of the particles in both the high energy and low energy regions in the framework of the theory. But a particular attention will be devoted to the behaviour of the particles in the IR region where the particles are nonperturbative with a relatively large running coupling. This is intended to enhance the current understanding of nonperturbative QCD. We will study several confining features including; glueball masses, QCD beta function $\beta(Q^2)$, the strong running coupling $\alpha_s(Q^2)$, QCD vacuum, vector, scalar and net confining potentials with their associated string tensions. We explore the effect of temperature on the models resulting into thermal deconfinement and restoration of chiral symmetry at higher temperatures beyond the critical temperature of the system. The scalar field that modify the DBI theory is related to tachyons, and it is carefully defined to produce Spontaneous Symmetry Breaking (SSB) of the system. This phenomenon will make it possible to investigate elementary particles, dual superconductivity, glueballs, glueball-meson mix and other phase transitions.

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Chapter 1

Introduction

This thesis is partly based on the journal papers [1, 2, 3, 4].

1.1 Background

There are four known fundamental forces in nature namely; electromagnetic, weak, strong and gravitational interactions that explain all physical occurrences in nature. The first three of these forces can be studied under the *Standard Model* (SM) theory, while gravitational interaction is treated under *General Relativity*. It is the goal of physicists to develop a theory that will unify all these forces under a single framework. The initial approach to the SM was to unify the electromagnetic and weak interactions introduced by Sheldon Glashow in 1961 [5]. Later in 1967 Steven Weinberg and Abdus Salam also introduced the Higgs mechanism into Glashow's electroweak theory to give it its current status [6, 7]. So far, there are some advances to unify all the interactions with *String theory* [8, 9, 10, 11]. Additionally, strong interaction is the strongest interaction among the four. It operates among quarks, antiquarks and gluons at the nuclear level. It keeps the nuclei together. Indeed, it is the reason the protons inside the nucleus do not repel each other due to electromagnetic interactions. The theory that gives an indepth description into this phenomenon is *Quantum Chromodynamics* (QCD)[12, 13, 14, 15].

QCD and Quantum Electrodynamics (QED) exhibit some similarities in terms of their nature, and how both theories are formulated based on perturbation [16, 17]. On the other hand, they show some differences in terms of strength, medium and dynamics of interactions. In QED, the concept of conserve charge is crucial while its equivalence

in QCD is the color charge introduced by Fritzsch, Gell-Mann and Leutwyler [18]. These classifications are better understood by examining their associated gauge groups. QED is associated with the abelian $U(1)$ gauge group, with two state electric charges (+ or -). On the flip side, QCD is related to the $SU(3)$ gauge group with three color charges (red, blue and green) or their anticolor charges (antired, antiblue and antigreen). Again, QED generates a *screening effect* which causes the net electric charge to decrease as inter-particle distance increases. On the other hand, QCD generates an *antiscreening effect* where the net color charge rather increases with increasing inter-particle distances. Quarks are fundamental particles which interact with each other through gluon exchanges. Thus, gluons are the gauge bosons that mediate the interactions and are the carriers of the color charges. In group theory, the quark is in the fundamental representation while the gluon is in the adjoint representation.

Phenomenologically, QCD has three principal features; color confinement, chiral symmetry (breaking and restoration) and asymptotic freedom which manifest differently in the infrared (IR) and the ultraviolet (UV) regimes of the theory. The initiators of the concept of asymptotic freedom are Gross, Wilczek and Politzer [16, 17]. They observed that the QCD running coupling $\alpha_s(Q^2)$ which determines strength of the interactions decrease with energy increase or higher momentum transfer Q . This can be interpreted to mean that, as the energy keeps increasing, the interaction between color particles keeps weakening until such a point, where they actually feel free from each other. This is where the idea of the asymptotic freedom was derived from. That notwithstanding, at lower energies the strength of $\alpha(Q^2)$ becomes so high to an extent that the color particles can be said to be confined inside bigger structures known to be hadrons [19].

The success of quark model was derived from hadron models [20, 21]. Currently, it has been well understood that hadrons are composites with different quark flavour combinations. Combination of three quarks form color neutral *baryons* whilst the combination of quark and antiquark (two quarks) form a colorless *meson*. There are eight known massless and flavourless color gluons with spin 1. The (anti)quarks are spin 1/2 fermionic particles. What is intriguing is the assignment of masses and charges to quarks, because it is not possible to measure the ' e/m ' for quarks. Because quarks do not exist as free particles (or asymptotic states). Despite this challenge, collision of electron-positron in the hadron channel and in the lepton channel have a good agreement with the assigned

masses and charges [22, 23, 24, 25]. Also, it affirms that quarks behave like free particles at very high energies or higher momentum transfers. The reason free quarks or any particle with color quantum number has never been observed in isolation despite all the brilliant efforts is due to *confinement* [26, 27]. In this thesis, we extend the study to various types of glueball masses (scalar, isoscalar and hybrid glueballs) [28, 29]. Unfortunately, pure glueballs are very hard to identify in *particle accelerators* because they easily mix with mesons to elude identification.

Even though QCD has received acceptance among the community of physicists, it has some aspects that needs to be exploited further for clearer understanding. For instance, the concepts of *color confinement* remains an open question that needs further investigations and comprehension. Hadron spectra and QCD phase transitions are also related concepts that needs further investigations and clearer understanding. *Glueball masses*, *dual superconductors* and *QCD monopole condensation*, which are byproducts of confinement, the nature and structure of hadron jets also remain opened. These areas and many other related gaps makes QCD theory an interesting field of study for both theoretical and experimental physicists. In view of theoretical physicists, all these short falls are due to *color confinement* and *nonperturbative* nature of QCD theory. At low momentum transfers, $\alpha_s(Q^2)$ becomes large making the application of perturbation theory unsuitable, unlike QED theory. This motivates the introduction of several other approaches such as; lattice QCD (lQCD), quasiparticle approach, QCD sum rule, phenomenological QCD ect. to study the behaviour of quarks and gluons in the IR regime. In view of experiment, the minuteness of the length scale ($\leq 10^{-18}\text{m}$) implies that larger accelerators and more sophisticated detectors needs to be built in order to be able to investigate matter of this nature.

This thesis is intended to contribute to explaining the limitations associated with QCD theory. We resort to string theory, where all the fundamental particles are described in terms of strings sitting on a background Dp-branes [30, 31, 32]. In this case, classification of particles and their masses are related to energy of the string oscillations. String theory was the original theory intended for nuclear matter [33, 34] in late 1960's before it was overtaken by the QCD theory in 1970's [16, 17]. The theory have since evolved, providing useful interventions in other areas of physics as well [35, 36, 37, 38, 39]. Recent probes establish that the string theory and QCD theory describe the same physics. It is

also expected to provide the background for the unification of all the fundamental forces [40, 8, 41, 42, 43]. Consequently, it is the hope for probing physics beyond the standard model. We developed theories by modifying the well-know DBI theory and analyse it at tachyonic vacuum to describe QCD phenomenologically. Based on these theories, we build various models that mimic the characteristics of QCD phenomenology. We were motivated by the characteristics of open strings at tachyonic vacuum and the natural relation between tachyon potential and color dielectric function. The nonperturbative nature of Dp -branes, Abelian dominance in low energy QCD, high symmetry among tachyon fields and the treatment of tachyons in Quantum Field Theory (QFT).

We introduce an intermediate tachyon field ϕ to modify the DBI action however, this field was carefully defined to mimic the behaviour of the usual Higgs field. The SM was used to predict the Higgs boson (H) in 1976 [44, 45, 46], it is responsible for generating fermion masses and other massive bosons in the interaction. It was later discovered in 2012 [47, 48] at LHC. As a result, we will develop theories and models using the scalar field ϕ in place of the Higgs field to generate mass. This kind of analogy was possible because the scalar field behaves like a tachyonic field where the usual Higgs mechanism proceeds via tachyon condensation [49, 50]. Additionally, Higgs and tachyonic fields have some similarities; they are both considered unstable, associated with rapid decay, high symmetry and negative square mass among others.

1.1.1 Confinement

Confinement is so far known to be the result of the large magnitude of QCD running coupling at lower momentum transfers. It has a corresponding linear potential growth between quark and antiquark pairs. To verify this behaviour, one needs to calculate the potential in the low energy regime (long distance or the IR regime) using phenomenological approaches. Since the behaviour in the low energy region are nonperturbative in nature, it is not tractable by the QCD theory. In this regime, perturbation expansions such as the ones employed in Feynman diagrams are not suitable. However, in 1974 Wilson [26] showed that there is confinement in gauge theories for strong coupling limits using lattice regularization approach. Creutz [51] and Wilson [52] adopted another approach in 1980, by using Monte Carlo analysis to investigate QCD lattice in four dimensional spacetime, and it showed a good agreement with confinement as well. Due to the complexity of

tracking confinement of quarks and gluons with the fairly accepted QCD theory, many other approaches including phenomenological models have been exploited to address this problem.

In this thesis, we develop phenomenological models by approximating the non-Abelian gauge field responsible for color confinement with an Abelian one coupled with a color dielectric function $G(\phi)$. Due to gluon confinement the QCD vacuum behaves like a dielectric medium [53, 54]. This is the basics for developing several phenomenological models, including MIT bag model [20], SLAC bag model [55], Cornell potential for heavy quarks [56] and many soliton models [57, 58, 59] which are used to describe hadron spectroscopy. The Abelian projection of QCD theory was proposed by 't Hooft to justify magnetic monopole condensation in QCD vacuum leading to *dual superconductivity* [60, 61] and he also suggested 'infrared Abelian dominance'. Fast forward, it has also been shown that about 92% of the QCD string tension is Abelian [62], so an Abelian approximation will not be out of place. Further studies of Abelian dominance for color confinement in SU(2) and SU(3) lattice QCD can be found in Ref.[63, 64]. An interested reader can also refer to Ref.[65] for more recent results on Abelian dominance in QCD theory. We further introduce mass into the model to study scalar and vector confinement similar to what is observed in [66].

Despite the establishment that strong interactions are non-Abelian in nature, confining potentials are yet to be computed through this formalism. This is the reason phenomenological models have become prominent in QCD theory to fill in the gaps. For instance, the linear and Cornell potentials have since proven to be vital in describing strong interactions. Further studies of high energy hadron collisions have also shown that bound states of heavy quarks, glueballs, glueball-meson mix and quark-gluon-plasma (QGP) phase transitions are all relevant in understanding QCD. As a result, several experimental groups such as CLEO, LEP, CDF, D0, Belle and NA5 have produced data and currently ongoing experiments at BaBar, ATLAS, CMS and LHCb are producing and are expected to produce more precise data in the near future [67, 68, 69, 70, 73, 71, 72].

1.1.2 QCD Vacuum

The QCD vacuum condensates are usually presented as averages of operators obtained in the nonperturbative regime of the theory. They play a significant role in un-

derstanding hadron structure in the low energy regimes. For instance, quark condensates are baryon masses while gluon condensates are linked to glueball masses (bound states of gluons). Hence, condensates are believed to be dynamically generated mass scales in QCD. It is known that the QCD vacuum has nontrivial structure, different from what is seen in QED theory. The nontriviality of the condensate is similar to what is observed in condensed matter physics. This behaviour is attributed to the nonperturbative nature of the vacuum because of the non zero fluctuations of the gluon and the quark fields [74, 75]. Recent review on this subject can be found in [76].

A vacuum quark condensate is normally represented as $\langle 0|\bar{q}q|0\rangle$, where $|0\rangle$ is a hadron-less ground state. A non zero value of this condensate is the bases for dynamical chiral symmetry breaking (DCSB), an important concept in QCD which is linked to the 'bare' quark mass [77, 78, 79, 80, 81], one can also see Refs.[82, 83, 84] for more reviews. The DCSB also leads to the formation of *constituent quark masses* resulting into *color confinement*, *glueball masses* and other confinement properties such as QCD monopole condensation and *dual superconductivity*. To deal with condensate in QCD which lies in the low energy nonperturbative regime of the theory, phenomenologically, chiral perturbation theory (χ PT) [85] is inevitable. The condensate in χ PT is a low energy constituent with hadrons as fundamental degrees of freedom. Adding mass m_q term to the χ PT Lagrangian breaks the chiral symmetry explicitly resulting into an introduction of a parameter B_0 to the leading term which is identified as the quark condensate beside the pion decay constant f_π [86].

The experimental observation of QCD vacuum condensation dates back to pre-QCD era 1950-1973 to post-QCD after 1974, and still attracts attention among physicists. Because of its role in explaining color confinement and chiral symmetry breaking which form crucial part of strong interactions. Since color particles are not expected to be seen directly in nature, its composites in the form of hadrons (baryons and mesons) are observed experimentally. Meanwhile, QCD sum rule play an important role in describing the condensates theoretically, exploring gauge invariant formalisms [87, 88]. However, it is more reliable in the intermediate energy levels, so Vacuum Correlator Method (VCM) is resorted for a comprehensive understanding in the low energy region where color confinement is more pronounced [89, 90, 91].

1.1.3 QCD Phase Transition

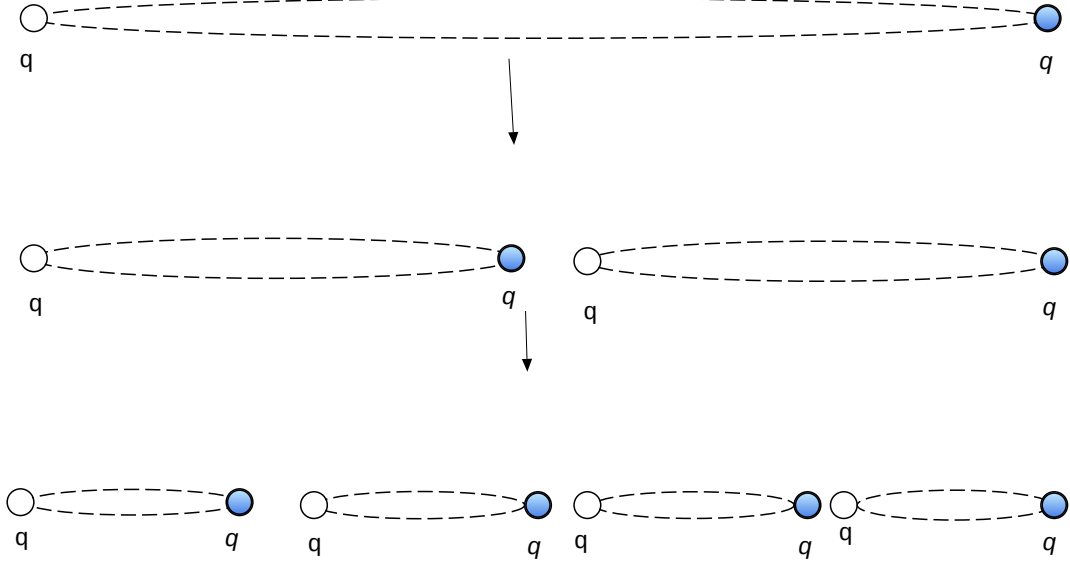
The QCD phase at an extremely high temperature and density has an important significance in both elementary particle physics and cosmology. The vacuum consists of square masses of quarks m_q^2 and/or gluons m_ϕ^2 depending on whether you are using unquenched and/or quenched approximations. In cosmology, it can be used to explain the evolution of the universe at extremely higher temperatures when matter becomes super dense and hadrons dissolve into a 'soup' of their constituents — quarks and gluons. At such superdense matter regimes the quarks become very close and asymptotically free, so the quarks are no longer confined into hadrons. It is important to note that the energy density of relativistic particles ρ , is related to temperature as $\rho \propto T^4$, so the density increases with temperature. This phase of matter was proposed [92, 93] after the asymptotic free [16, 17] nature of the QCD theory [18] was determined. Currently, almost all the accepted models for the cooling of the universe are based on phase transitions, either first or second order of spontaneous symmetry breaking of fundamental interactions [94, 95, 96]. On the other hand, the standard model (SM) of elementary particle physics suggests two of such phase transitions [96]. It shows an electroweak symmetry (EW) breaking at temperatures in the order of $T \sim 100$ GeV which generates mass for elementary particles such as quarks. It is also associated with the observed baryon-number-violation of the universe [97]. This transition is determined to be an analytical crossover in lattice simulations [98].

Spontaneous chiral symmetry breaking is next, it is known to occur at temperatures in the order of $T \sim 200$ MeV. Nevertheless, we have hadronic matter below this temperature and above it, we have an expected transit into the QGP phase. This phase is also important in understanding the evolution of the early universe. Some references on this subject using nonperturbative lattice QCD models can be found in Ref.[99, 100]. Particularly, the baryon chemical potential is expected to be smaller than the usual hadron mass, $\mu_B \approx 45$ MeV at $\sqrt{s_{NN}} = 200$ GeV [101], and almost vanishing $\mu_B \approx 0$ GeV in the early universe. There is strong evidence that confinement of quarks into hadrons are low energy phenomena [99, 102, 103] but signals that QCD phase transition is a crossover. Numerically, it has been established that at $\mu_B = 0$ GeV the two phase transitions possibly coincide i.e. deconfinement and chiral symmetry restoration [104].

This transitions are basically driven by two essential thermodynamic parameters

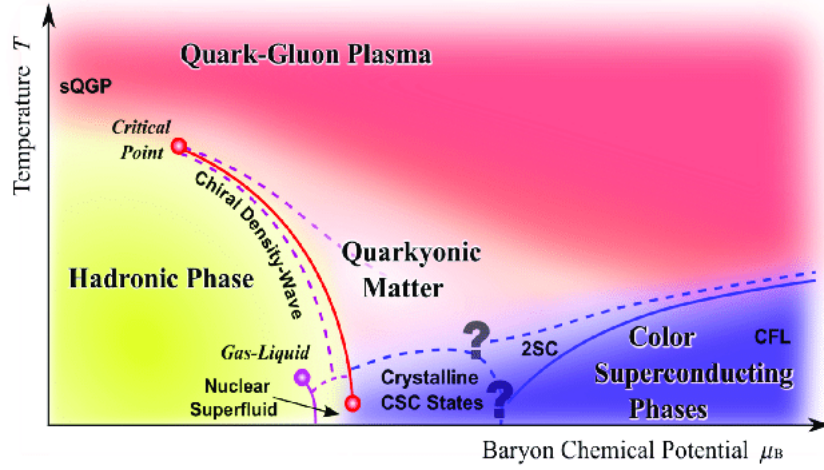
i.e. temperature and baryonic potential. At low values of these parameters the quarks and gluons are confined into hadron. When these parameters are increased beyond their critical values, we have a deconfinement phase. These phases are better understood using *Nambu-Goto string or the chromoelectric flux tube picture* models. Where a thin one dimensional flux tube is formed between a quark and an antiquark to keep them in a confined state [105, 106]. At the confinement phase or the fundamental state, the string remains intact, while at the deconfined phase or the excited state, the string breaks leading to *hadronization* i.e. a process where new quark-antiquark pairs are produced from the initial ones. This process continues when the thermodynamic parameters keep increasing until the quarks and gluons start behaving like free particles.

Figure 1.1: QCD String Breaking and hadronization.



A simplified picture of hadronization taking place at extreme conditions, the quarks are represented with q and the antiquark with $\bar{q}$. Generally, the particles experience asymptotic freedom when they are very close to each other. However, increasing the distance between the particles leads to an IR dominance at a point leading to confinement. When the distance between the particles are increased beyond the IR limit, hadronization and screening sets in, as seen in the diagram above (at this point we assume that all thermodynamic parameters remain constant). On the other hand, when thermodynamic parameters such as temperature and baryon density of the system are increased beyond the IR limit (here separation distance between the particles remains constant), hadronization and deconfinement are observed in different ways. We observe thermal deconfinement when the critical temperature is reached or exceeded whilst we obtain deconfined matter regime leading to color superconductivity when the critical baryon density is reached or exceeded.

Figure 1.2: QCD Phase Diagram.



The QCD phase diagram is still under studies for better understanding experimentally [107, 108, 109].

Thermal QCD have been approached in several ways including Wilson and Polyakov loop corrections [110, 111, 112, 113]. The main problem associated with this subject is the correct behaviour of the string tension (σ) at all temperatures i.e. $T = 0$, $T < T_c$, $T = T_c$ and $T > T_c$ compared to the results obtained from lattice simulations. The expected behaviour is strong σ at $T = 0$, sharply decreasing σ at $T < T_c$, vanishing at $T = T_c$ and slowly decreasing at $T > T_c$ [13, 14, 113, 114].

1.1.4 Structure of the Thesis

This thesis is partly based on journal papers [1, 2, 3, 4]. It is organised as follows: In Chap. 2 we reviewed the following; The Quark Model, current understanding of the Classical QCD Theory, Phenomenological QCD Models, Holographic QCD, Lattice QCD, Glueballs and Maxwell Equations. Chap. 3 marks the beginning of our original contributions to the subject under study. In this chapter, we develop the theories that will form the bases for the thesis. Here, we present a brief review of Tachyons and D p -branes followed by Modification of DBI Action, Bosonic Opened Strings at Tachyonic Vacuum and Modified Gauge Theories with Fermion Couplings. Chap. 4 and subsequent chapters that follow it constitute the models that are built from the theories developed in Chap. 3. In Chap. 4 we investigate Confinement of Glueballs, Chaps. 5 and 6 are made up of Confinement of Fermions and Confinement of Fermions at Finite Temperature respectively. We proceed to analyse Glueballs and Dual Superconductivity in Chap. 7 and we present our Findings, Conclusions and Projections in Chap. 8.

Chapter 2

Review

The objective of this chapter is to provide some useful highlights to the basics of QCD theory for none expert readers. This is intended to prepare the mind of the reader to better appreciate the content of this thesis. We will deliberately focus on phenomenological models in line with the objectives and the motivations of the thesis. The unsatisfactory understanding of *color confinement* from QCD theory has motivated various phenomenological models to provide explanations to complement the QCD theory. Though, most of these models lack a direct connection to QCD theory, they have been largely successful in describing the properties of hadrons. Some of these models include MIT bag [20], SLAC bag [55], Cornell potential for heavy quarks [56], and many soliton models [57, 58, 59].

We will first of all present the classical description of QCD because it carries the real quark and gluon degrees of freedom for which the QCD theory was built on. Again, as part of this chapter we will present a brief review of other models that may be used in subsequent chapters to build our intended models.

2.1 The Quark Model

In 1964 Gell-Mann [116, 117] and Zweig [118] proposed a theoretical way of classifying hadrons, generally refereed to as *the quark model*. The background of this model is to use symmetry arguments that govern the group representations on which the Lagrangian of the strong interaction is invariant. The hadrons are composed according to their fundamental representations, group symmetry and their quantum numbers emanat-

ing from their conserve charges. These fundamental particles were initially considered as mathematical tools named as quarks by Gell-Mann. They were eventually discovered through a series of an inelastic collisions of electron and nucleon from 1967 to 1973 at Massachusetts Institute of Technology (MIT) and Stanford Linear Accelerator Center (SLAC) [119, 120, 121]. This led to the award of Nobel Prize to Jerome Friedman and Henry Kendall of MIT and Richard Taylor of SLAC in 1990 for discovering the internal structure of hadrons.

One of the symmetries in strong interaction is $SU(2)$ -isospin symmetry that exist between proton and neutrons. This symmetry was proposed by Heisenberg long before the discovery of quark model. Since protons and neutrons are found to have the same mass, their strength of interaction is almost the same without electromagnetic effects. In view of this, the nucleon was said to have isospin $I = 1/2$, where I is (strong) isospin, consistent with two dimensional $(2I + 1)$ representation of $SU(2)$. Thus, in strong interactions, proton and neutron are two different representation of similar object (the nucleon), which only differ by an electric charge. So, introducing electromagnetic field into the Lagrangian for strong interaction breaks the $SU(2)$ symmetry and change its direction on the isospin space onto I_3 . That notwithstanding, the isospin generators τ_1, τ_2, τ_3 do not commute with others in different spatial directions, so only one component can be measured at a time and the third component τ_3 is choosing by convention. Hence, $I_3 = 1/2$ represents proton while $I_3 = -1/2$ represents neutron. Also, pions (π^+, π^0, π^-) have $I = 1$ belonging to the adjoint representation of $SU(2)$, this defines a multiplet of the symmetry group. Its projection on I_3 are 1, 0, and -1 , consequently, particles are classified according to their representations on $SU(2)$ Lie groups [123, 124, 125].

This symmetry group was acceptable at the time because hadrons were thought to be made up of only up (u) and down (d) quarks. Nonetheless, these quarks are still perceived to be the only ones present in the nuclei structure even though unknown (see Ref.[122] for the history on the discovery of quarks). Fast forward, this proposal became questionable when 'strange' hadrons were discovered in 1947 known as kaons ($K^\pm, K^0, \bar{K}^0, \dots$) and hyperons ($\Sigma^\pm, \Sigma^0, \dots$) in cosmic rays and accelerators. Unfortunately, these hadrons did not follow any of the known decay processes at the time. An attempt to explain this strange behaviour was made when new quantum number known as 'strangeness' which is conserved during strong interaction was proposed in 1950's. Indeed,

new conserve charge calls for new symmetry in the Lagrangian involved, so the SU(2)-isospin symmetry was extended to SU(3) to account for it. *Behold the quark model was born with an SU(3)-flavour symmetry.* Additionally, hadrons were identified as composite structures made up of elementary triplets of spin-1/2 quarks. The three quark 'flavors', namely, *up*, *down* and *strange* (*s*) are associated with the quantum numbers; (strong) isospin (I), strangeness (S), baryon number (B), (strong) hypercharge (Y) and electric charge (Q). Mesons composed of quark and antiquark ($q\bar{q}$) lies in the SU(3) representation as

$$q\bar{q} \Rightarrow 3 \otimes \bar{3} = 1 \oplus 8, \quad (2.1)$$

while baryon composed of three (anti)quarks are represented as

$$qqq \Rightarrow 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10. \quad (2.2)$$

Now, we present the known properties of quarks on table 2.1 including their projected masses.

Table 2.1: Properties of Quarks.

Light Quarks							
Quarks	I	I_3	S	B	$Y = B + S$	Q	Mass (MeV)
u	1/2	1/2	0	1/3	1/3	2/3	~ 2.8
d	1/2	-1/2	0	1/3	1/3	-1/3	~ 5.2
s	0	0	-1	1/3	-2/3	-1/3	~ 104
Heavy Quarks							
Quarks	I	I_3	Q	Charm	Beauty	Truth	Mass (GeV)
c	0	0	2/3	1	0	0	~ 1.3
b	0	0	-1/3	0	-1	0	~ 4.2
t	0	0	2/3	0	0	1	~ 173

It is worth noting that SU(3)-flavor symmetry is not an exact symmetry of the QCD Lagrangian. The mass differences between u, d and s quarks breaks the symmetry. Typically, the entries in table 2.2 seem the hadron model violates some fundamental physics, considering all the degrees of freedom necessary. It appears some of the wave

functions associated with the hadrons in table 2.2 violates the Pauli's exclusion principle. Bosons (fermions) are known to have (anti)symmetric wave functions. For instance, Δ^{++} is a fermion which seem to have a symmetric wave function contrary to the known fundamental principle. This led to the introduction of *color degrees of freedom* which was later confirmed by experiments to cure the anomaly. Therefore, *color confinement* accounts for the color degrees of freedom which produces the antisymmetric characteristics of the wave function to fulfil the Pauli's principle. This gives SU(3)-color symmetry the prominence in gauge field theory responsible for describing QCD theory [123, 124, 125].

Table 2.2: Properties of Hadrons.

Mesons				
Hadron	Charge	Spin	Strangeness	Examples
$u\bar{d}$	1	0	0	π^+
$\bar{u}d$	-1	0	0	π^-
$u\bar{u}, d\bar{d}$	0	0	0	π^0
$u\bar{s}$	1	0	1	K^+
$\bar{u}s$	-1	0	-1	K^-
$d\bar{s}$	-1	0	1	K^0
$u\bar{u}, d\bar{d}, s\bar{s}$	0	0	0	η
Baryons				
Quarks	Charge	Spin	Strangeness	Examples
uud	1	1/2	0	p
udd	0	1/2	0	n
$uuu,$	2	3/2	0	Δ^{++}
uud	1	3/2	0	Δ^+
udd	0	3/2	0	Δ^0
ddd	-1	3/2	0	Δ^-
uus	1	1/2	-1	Σ^{*+}, Σ^+
uds	0	1/2	-1	$\Sigma^{*0}, \Sigma^0, \Lambda^0, \Lambda$
dds	-1	1/2	-1	Σ^{*-}, Σ^-
uss	0	1/2	-2	Ξ^{*0}, Ξ^0
dss	-1	1/2	-2	Ξ^{*-}, Ξ^-
sss	-1	2/3	-3	Ω^-

2.2 Bases of QCD Theory

From the above discussions we have established that the dynamics of mesons and baryons are characterised by the theory of quarks and gluons, generally referred to as Quantum Chromodynamics (QCD) [126, 127]. The objective of this theory was to sat-

isfy the hadronic phenomena with quark and gluon degrees of freedom. This theory has been largely successful due to the asymptotically free nature of the theory in high energy processes. Perturbation theory works well in this energy regime [128]. On the contrary, the theory shows some short falls at the low energy regime where QCD shows nonperturbative behaviour. Nonetheless, with the benefit of various experimental data, much is known about the behaviour of the theory in the low energy nonperturbative regime. For instance, properties such as: *nonperturbative vacuum structure* [129], *existence of the Nambu-Goldstone (NG) bosons and dynamical breaking of chiral symmetry* [79, 130], *$U(1)_A$ anomaly* [131, 132], *explicit $SU(3)_V$ breaking* [85, 133], *success of the constituent quark model* [134, 135, 136], *Okubo-Zweig-Iizuka (OZI) rule hadrons* [137] have been established. These properties have been separately investigated but what remain an issue is how to unify them under one theory.

2.2.1 The QCD Lagrangian

The classical chiral effective QCD Lagrangian with non-Abelian gauge group, $SU(3)$ color symmetry, is expressed as

$$\mathcal{L}_{QCD}^d = \sum_{f=1}^{N_f} \bar{\psi}_f (i\gamma_\mu D^\mu - m_f) \psi_f - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}. \quad (2.3)$$

Here, ψ_f and $\bar{\psi}_f = \psi_f^\dagger \gamma^0$ represent the quark and antiquark fields with three colors (red, blue and green) and N_f flavours $\psi_f = (u, d, s, \dots)$, m_f is the mass matrix for current quarks $m_f = \text{diag}(m_u, m_d, m_s, \dots)$, $D_\mu \equiv \partial_\mu - ig t^a A_\mu^a$ is the covariant derivative with colored gauge field $A_\mu^a (a = 1 \sim 8)$ and γ_μ is the Dirac γ -matrices. Here, g is the strong coupling constant and t^a are the $SU(3)$ -color gauge algebra generators that follow the relations

$$[t^a, t^b] = i f_{abc} t^c \quad \text{and} \quad \text{tr}(t^a t^b) = \delta^{ab}/2. \quad (2.4)$$

Where $A_\mu = A_\mu^a t_a$ and f_{abc} is the structure constant of the associated gauge group algebra, in this case, $SU(3)$. And $F_{\mu\nu}^a$ is the field strength which can be expressed as $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$. Generally, for $SU(N_c)$, a takes in the values $a = 1, 2, 3, \dots, N_c^2 - 1$.

It is important to note that the QCD Lagrangian is invariant under Lorentz transformation, space and time reversal and local gauge symmetries,

$$\Omega(x^\nu) = e^{i\theta^a(x^\nu) t_a}, \quad (2.5)$$

here, $\theta^a(x^\nu)$ is the infinitesimal parameter that depends on the space-time four vector x^ν . Therefore, quark-antiquark and gluon follow the transformations

$$\psi(x^\nu) \rightarrow \Omega(x^\nu)\psi(x^\nu) \quad (\text{fundamental representation}) \quad , \quad (2.6)$$

$$\bar{\psi}(x^\nu) \rightarrow \bar{\psi}(x^\nu)\Omega^\dagger(x^\nu) \quad (\text{conjugate representation}) \quad , \quad (2.7)$$

and

$$A_\mu(x^\nu) \rightarrow \Omega(x^\nu) \left(A_\mu(x^\nu) + \frac{i}{g} \partial_\mu \right) \Omega^\dagger \quad (\text{adjoint representation}) \quad , \quad (2.8)$$

respectively. Also, one can obtain a local gauge invariance using similar procedure as the successful Abelian technique used in QED. The major difference between the Abelian and non-Abelian gauge groups is the complexity of SU(3) algebra and occurrence of interaction among the associated gauge bosons. At infinitesimal level, we can expand $\Omega(x^\nu) \simeq 1 + i\theta^a(x^\nu)t_a$ and the expressions in Eqs.(2.6), (2.7) and (2.8) become

$$\psi(x^\nu) \rightarrow \psi(x^\nu) + i\theta^a(x^\nu)t_a\psi(x^\nu), \quad (2.9)$$

$$\bar{\psi}(x^\nu) \rightarrow \bar{\psi}(x^\nu) + i\theta^a(x^\nu)(-t_a^*)\bar{\psi}(x^\nu), \quad (2.10)$$

and

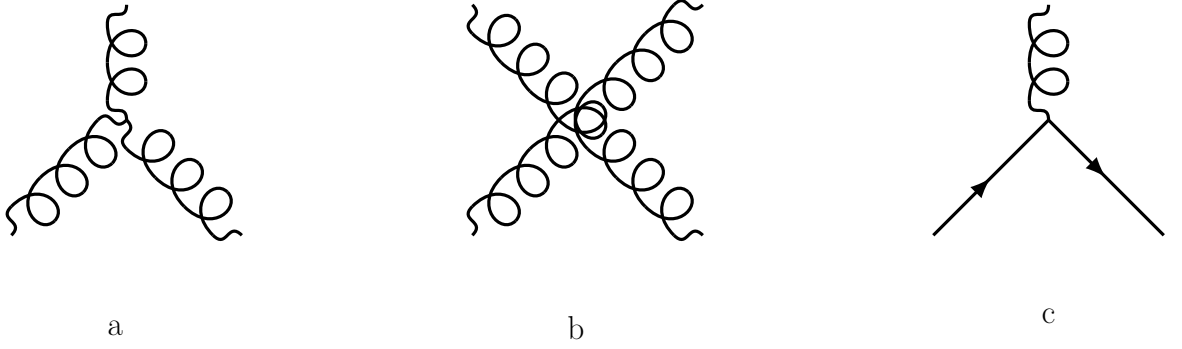
$$A_\mu^c(x^\nu) \rightarrow A_\mu^c(x^\nu) + \theta^a(x^\nu)f_{ab}^c A_\mu^b(x^\nu) + \frac{1}{g}\partial_\mu\theta^c(x^\nu) \quad (2.11)$$

respectively. Thus, adding terms of higher orders of $F_{\mu\nu}^a$ and ψ do not destroy Lorentz, space-time reversal and local gauge invariances. Consequently, the major difference between QED and QCD in terms of their gauge bosons is that, in QCD we have gluon self-interactions contrary to QED. In view of this, gauge contributions such as;

- three-gluon ($3 - g$) self-interaction: $A_\mu^a A_\nu^b A_\rho^c$
- four-gluon ($4 - g$) self-interaction: $A_\mu^a A_\nu^b A_\rho^c A_\sigma^d$

are possible under renormalizable QCD theory.

Figure 2.1: Feynman diagrams for gluon g self-interactions and quark-antiquark and gluon $\bar{\psi}^a A_\mu^b \psi^c$ ($\bar{q}qg$) vertices.



The diagrams are in the order: (a) 3 – g vertex, (b) 4 – g vertex and (c) $\bar{q}qg$ vertex.

Moreover, if perturbation theory in terms of g is to be carried out at the quantum level, we are required to introduce, gauge fixing terms and their corresponding ghost terms obtained from the Faddeev-Popov approach [138]. Additionally, due to regularization/renormalization of the theory, a mass scale μ , also called *renormalization scale* is introduced [139] to achieve the desire result. The QCD β -function can then be expressed in terms of the μ -dependence renormalized *strong coupling constant* g as

$$\mu \frac{d}{d\mu} g(\mu) = \beta(g). \quad (2.12)$$

Considering that g is sufficiently small, the β -function can be calculated from perturbation theory as

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2} g^3 - \frac{\beta_1}{(4\pi)^4} g^5 + \dots; \quad (2.13)$$

where

$$\beta_0 = 11 - \frac{2}{3} N_f \quad \text{and} \quad \beta_1 = 102 - \frac{38}{3} N_f, \quad (2.14)$$

here, N_f is the number of active flavours present. The β -function satisfies

- for $\beta(g) < 0$; g decreases with μ
- for $\beta(g) > 0$; g increases with μ
- for $\beta(g) = 0$; g is independent of μ (conformal theory).

Again, the *strong running coupling* α_s can be expressed in terms of g and μ as

$$\begin{aligned}\alpha_s(\mu) &= \frac{g^2(\mu)}{4\pi} \\ &= \frac{12\pi}{(33 - 2N_f) \ln(\mu^2/\Lambda_{QCD}^2)} \left[1 - \frac{6(153 - 19N_f) \ln[\ln(\mu^2/\Lambda_{QCD}^2)]}{(33 - 2N_f)^2 \ln(\mu^2/\Lambda_{QCD}^2)} \right] + \dots; \quad (2.15)\end{aligned}$$

where Λ_{QCD} is the QCD scale parameter that control how α_s varies with μ . It is also a function of the *minimum subtraction* ($\overline{MS}$) *scheme* [140] and the active flavor number, N_f . Some values obtained from high energy processes [141] are

$$\begin{aligned}\Lambda_{\overline{MS}}^{(3)} &= (290 \pm 30) \text{ MeV} \\ \Lambda_{\overline{MS}}^{(4)} &= (220 \pm 90) \text{ MeV} \\ \Lambda_{\overline{MS}}^{(5)} &= (140 \pm 60) \text{ MeV};\end{aligned} \quad (2.16)$$

where the superscript is used to represent the number of flavors. From Eq.(2.15), $\alpha_s(\mu)$ decreases logarithmically as μ increases, giving rise to the celebrated *asymptotic freedom* [16, 17] behaviour that revolutionized QCD theory.

2.2.2 Chiral Symmetry

The QCD Lagrangian Eq.(2.3) possess' many symmetries aside the local SU(3) gauge symmetry. Common among others are:

- SU(2) isospin symmetry : $m_u = m_d$
- SU(3) flavour symmetry : $m_u = m_d = m_s$
- $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry: $m_u = m_d = m_s = 0$;

the last point in the list will be discussed in detail. This subsection will be dedicated to discussing light quarks which are important particles in understanding the real world. To throw light on this subject and explore the associated physics, it is appropriate to examine the theoretical limit where the quark masses are considered zero. This limit is referred to as the *chiral limit* [142, 143]. In this limit, the spin of the quarks can either be in the direction of motion or in the opposite direction denoted by right-handed ψ_R and the left-handed ψ_L fields respectively. To achieve this objective, we can decompose Eq.(2.3) into

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{chiral} + \mathcal{L}_{QCD}^{mass}; \quad (2.17)$$

where

$$\begin{aligned}\mathcal{L}_{QCD}^{chiral} &= -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \sum_f^{N_f} \bar{\psi}_f(i\gamma^\mu D_\mu)\psi_f \quad \text{and} \\ \mathcal{L}_{QCD}^{mass} &= -\sum_f^{N_f} \bar{\psi}_f(m_f)\psi_f.\end{aligned}\tag{2.18}$$

In the chiral limit $m_f = \text{diag}(m_u, m_d, m_s) = 0$, so $\mathcal{L}_{QCD}^{mass}$ can be ignored from the subsequent analysis. As a result, the Dirac spinors can be decomposed into the right-handed and the left-handed components as

$$\psi = \psi_R + \psi_L, \tag{2.19}$$

corresponding to

$$\psi_R = \frac{1 + \gamma_5}{2}\psi \quad \text{and} \quad \psi_L = \frac{1 - \gamma_5}{2}\psi. \tag{2.20}$$

The Lagrangian Eq.(2.18) can be expressed in terms of ψ_R and ψ_L with $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, which will be invariant under the transformations

$$\psi_L \rightarrow \psi'_L = U_L\psi_L \quad \text{and} \quad \psi_R \rightarrow \psi'_R = U_R\psi_R; \tag{2.21}$$

where $U_{L,R}$ are $N_f \times N_f$ unitary matrices. Generally, this symmetry is referred to as the *chiral symmetry*, represented as $U(N_f)_L \times U(N_f)_R$. Also, the spinor fields $\psi_R(\psi_L)$ are the eigenstates of the chirality operator with eigenvalues $1(-1)$. Moreover, the helicity h of massless quarks as discussed here can be expressed as

$$h = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|}; \tag{2.22}$$

where $\vec{S}$ is the spin and $\vec{p}$ is the corresponding momentum. In terms of Eqs.(2.19) and (2.20) we can decompose the chiral Lagrangian as

$$\mathcal{L}_{QCD}^{chiral} = \mathcal{L}_{QCD}^R(\psi_R, A_\mu) + \mathcal{L}_{QCD}^L(\psi_L, A_\mu), \tag{2.23}$$

therefore, the right and the left handed fields appear decoupled in the *chiral limit*. For three quark flavours $N_f = 3$, we can express $U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_R \times U(1)_L$. For the $U(1)$ parts, one is associated with baryon number ($U(1)_V$) and the other to an axial symmetry ($U(1)_A$), which are anomalous and will not be discussed here [144, 145]. Explicitly,

$$\begin{aligned}U(1)_R &= e^{i\alpha_R} \quad , \quad U(1)_L = e^{i\alpha_L}, \\ \psi_R &\rightarrow e^{i\theta_R^a \lambda^a} \psi_R \quad : \quad U(3)_R \quad \text{and} \quad \psi_L \rightarrow e^{i\theta_L^a \lambda^a} \psi_L \quad : \quad U(3)_L.\end{aligned}\tag{2.24}$$

In accordance with the Noether's theorem, the $SU(3)_L \times SU(3)_R$ chiral symmetry is associated with the conserve currents

$$J_{L\mu}^a = \bar{\psi}_L \gamma_\mu t^a \psi_L \quad \text{and} \quad J_{R\mu}^a = \bar{\psi}_R \gamma_\mu t^a \psi_R. \quad (2.25)$$

However, we can construct an axial ($J_A^{a,\mu}$) and vector ($J_V^{a,\mu}$) currents from linear combinations of the above equations

$$J_V^{a,\mu} = J_R^{a,\mu} + J_L^{a,\mu} = \bar{\psi} \gamma^\mu t^a \psi \quad \text{and} \quad J_A^{a,\mu} = J_R^{a,\mu} - J_L^{a,\mu} = \bar{\psi} \gamma_\mu \gamma_5 t^a \psi, \quad (2.26)$$

consequently, $\partial_\mu J_V^{a,\mu} = 0$, $\partial_\mu J_A^{a,\mu} = 0$ and their associated charges can be derived and denoted by Q_a and Q_{a5} respectively. From the afore discussions, we can associate the chiral symmetry $SU(3)_L \times SU(3)_R$ to the vector and axial symmetries as $SU(3)_V \times SU(3)_A$.

In conclusion, even though we treated the chiral symmetry based on $N_f = 3$ active flavours, same holds for $N_f = 2$ active flavours as well. When mass is introduced into the system, the *chiral symmetry breaks down spontaneously* leading to the release of some mesons and subsequently, *color confinement* ensue. For instance, in $N_f = 3$ we have

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V, \quad (2.27)$$

leading to the release of eight mesons; $\pi^\pm$, π^0 , $K^\pm$, $\bar{K}^0$, K^0 and η . For $N_f = 2$ we get

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V, \quad (2.28)$$

resulting into three pions; $\pi^\pm$, and π^0 [146].

2.3 Phenomenological Models

Due to unsatisfactory theories to properly understand *color confinement*, it has become prudent to introduce different phenomenological models to offer some explanations to fill in the gap. Even though some of these models do not have proper linkage to the QCD equations, they have shown some success in predicting hadronic properties in one way or the other. The bag models and the potential models are typical examples among others. In this section we will review some selected phenomenological models and discuss their relevance to *color confinement*.

2.3.1 The Bag model

The foremost confinement models were studied using relativistic bag model [20, 147]. The bag model was initiated to help physicists to liberate the quark from within the hadron for theoretical analysis when they were faced with the challenge. It is known from theoretical field theory investigations that the physical QCD vacuum, composed of microscopic structures which is seen as a 'normal phase' aside hadrons, do not provide an explanation to the propagation of quark and gluon fields. Despite the Lorentz invariant property of the vacuum, it shows a strange behaviour compared to hadronic constituent fields. Small domains of different phases like a bubble with the size of a hadron are expected to form in the vacuum. Quark and gluon fields are then propagated freely inside these bubbles. Hadrons can then be seen as small domains in the vacuum with quarks and gluons as their constituents — detailed description of the QCD vacuum is discussed in Sec. 1.1.2 —. This is known as the bag and the boundary between the bag and the vacuum phase is impermeable by the color fields. This impermeability is expressed in terms of the boundary conditions of the color fields [148]. Here, the gluon electric fields $E_i (i = 1, 2, 3, \dots, 8.)$ are tangential to the surface whilst the gluon magnetic fields B_i are normal to the surface. Hence, there is no net gluon energy flux through the surface of the hadron domain space. Inside the bag, the dynamics of the particles can be studied locally using QCD field equations. Again, in the hadronic phase, quarks and gluons are in a confined state due to the nature of the gluon gauge field.

The bag model is treated relativistically and the hadrons under this model framework are seen as a deformable droplets whose shapes are determined dynamically. Basically, in the bag model, the hadron is seen as spherical bag whilst the quarks within it are represented by the Dirac fields in a confined state within the volume of the bag with a finite radius equivalent to the size of a hadron ~ 1 fm. In this model, u and d quarks are of a particular significance because of their small masses (~ 10 MeV). That notwithstanding, a bag of these light quarks are not suitable because they exert pressure on the surface of the bag due to the freely moving quarks and gluons inside it. However, the model was used to successfully study light hadron spectroscopy in uds sector. A remedy to this problem was proposed by Nikolay Bogoliubov. His approach was to give the quark fields an enormous mass, intended to confine the quarks and restrict their movement within the bag. Upon first thought, this approach seem to violate the *asymptotic*

freedom behaviour associated with QCD theory when the quarks come very close to each other. He dealt with this violation by confining the quarks within a spherical cavity with radius R . Within this sphere, the quarks feel an attraction of fields with mass m . As a result, the quark masses are set to m and consequently, $m \rightarrow \infty$. This was how the Bogoliubov's bag model was conceived as a pioneer bag model. His model was christened 'a simple bag model', but was capable of giving some accurate predictions [147, 149].

Furthermore, MIT bag model is built up on the Bogoliubov's bag model to give an enhanced results. The major difference between the two models is the introduction of the bag constant B into the MIT bag model. This is to correct the internal pressure exerted by the light quarks, B serves as an external pressure term introduced into the model precisely to prevent the expansion. Hence, the model has Lorentz scalar term complimented by the bag pressure B . The bag constant has proven to be a viable correction by showing strong agreement with some predictions to masses of variety of quarks and hadrons. The Lagrangian density for the MIT bag model [150, 151] is given as

$$\mathcal{L} = \left[\frac{i}{2} \left(\bar{\psi} \gamma^\mu \partial_\mu \psi - (\partial_\mu \bar{\psi}) \gamma^\mu \psi \right) - B \right] \theta_v(x) - \frac{1}{2} \bar{\psi} \psi \Delta_s; \quad (2.29)$$

where ψ and $\bar{\psi}$ are the quark field and its conjugate, γ^μ is the Dirac γ -matrices [152]. Also, the step-function $\theta_v = \theta(R - r)$ has a property

$$\theta_v = \begin{cases} 1 & \text{if } r \leq R, \text{ (inside the bag) } \\ 0 & \text{if } r > R, \text{ (outside the bag) } \end{cases}, \quad (2.30)$$

and Δ_s is the surface delta function defined in terms of the step-function as

$$\frac{\partial \theta_v}{\partial x^\mu} = n_\mu \Delta_s \quad ; \quad \Delta_s = \delta(R - r). \quad (2.31)$$

Here, θ is the step-function, δ is the Dirac delta function, n_μ is the unit vector normal to the surface and R is the radius of the bag also,

$$\Delta_s = \begin{cases} 0 & \text{if } \theta_v = 1 \\ \infty & \text{if } \theta_v = 0 \end{cases}. \quad (2.32)$$

The inward vacuum pressure B acting on the surface of the bag balances the outward pressure generated by the quarks and gluons inside the bag. At the ground state, the pressure is isotropic. The bag gets deformed at the excited state and confinement is consequently,

imposed through boundary conditions. Additionally, the boundary is transparent to leptons and mediators such as photons γ , Z -boson and $W^\pm$ -bosons. These particles can be propagated in both the hadron and the vacuum phases normally. Indeed, in the model framework, the mass of the hadron depends on the number of quarks in the bag. It is important to add that, all the studies done under the bag model are based on the assumption that the bag is a static sphere.

2.3.2 Color Confinement and Color Dielectric Vacuum

Bag models were developed to explain why free color particles are not observed in nature. To explain this phenomena, we start with the classical electrodynamic Coulomb potential in a polarizable medium governed by

$$V(r) = \frac{e^2}{4\pi\epsilon r}; \quad (2.33)$$

where e is the electric charge, ϵ is the dielectric constant [53, 54] and $|\mathbf{r}| = r$ is the separation distance between two charges. However, in QED the electric charge is screened even in the vacuum where there is no polarizable material. Despite, the classical projection that $\epsilon = 1$ in the vacuum, $\epsilon > 1$ in QED vacuum due to the *screening effect* occasioned by the creation of virtual electron and positron pairs. By this, the net electric charge mediated by the photons grow weaker as the separation distance r increases [150]. In classical electrodynamics ϵ and μ (magnetic permeability) are related as $\epsilon\mu = 1/c^2$, in natural units, $c = 1$, so $\epsilon\mu = 1$ which is consistent with Lorentz invariant theories. As a result, for $\epsilon > 1$ we have a corresponding $\mu < 1$ in QED vacuum making it *diamagnetic*.

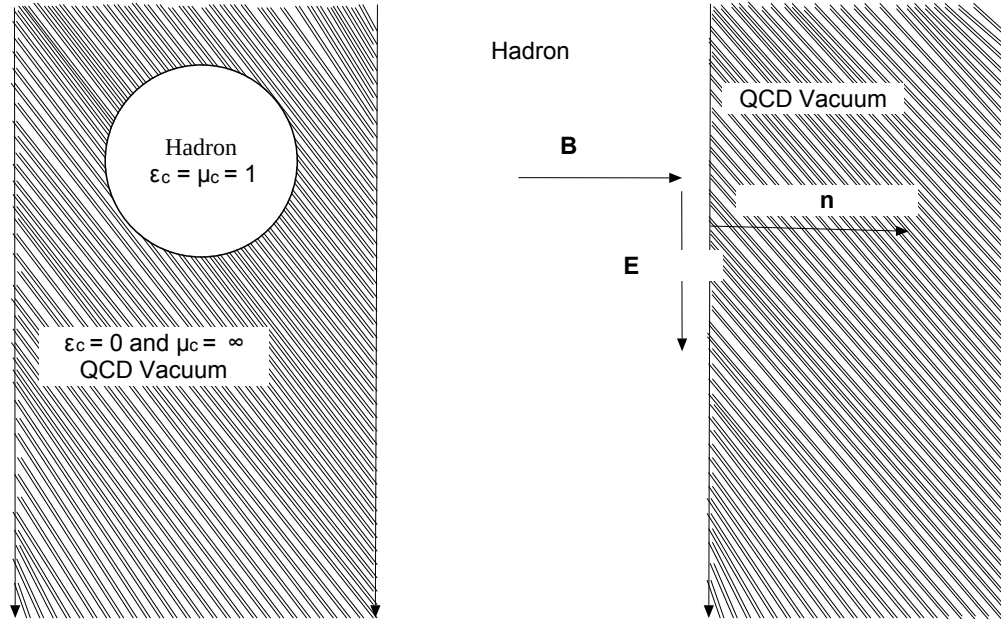
On the other hand, in QCD, quarks carry color charges through the exchange of color charged gluons. Hence, some screening behaviour similar to what happens in QED theory is expected, at least through *gluon cascade*, $g \rightarrow gg$, of the virtual gluons. Nonetheless, the color charge and the spin of gluon g leads to *color magnetization* of the medium thus, turn the medium *paramagnetic*. This property dominates the *diamagnetic* behaviour of $q\bar{q}$ pair expected to be created through $g \rightarrow q\bar{q}$ leading to a behaviour opposite to what is observed in QED. Accordingly, the color particles exhibit an *anti-screening* or *camouflage* effect. In the color medium $\epsilon_c < 1$ and $\mu_c > 1$, so the interactions between the color particles in the color dielectric medium are stronger for long distance particle separation r (where ϵ_c is a color dielectric constant). Therefore, the attraction

between the color particles increases with increasing r [153]. In sum, color dielectric constant follow the properties

$$\epsilon_c \rightarrow \begin{cases} 1 & \text{if } r \rightarrow 0 \\ 0 & \text{if } r \rightarrow \infty \end{cases}. \quad (2.34)$$

Additionally, as $r \rightarrow 0$, ϵ_c and μ_c approaches unity so, the interactions between the color particles become weaker, $\alpha_s \rightarrow 0$, and the particles become asymptotically free. Therefore, inside the hadron (bag) $\epsilon_c = 1$ corresponding to $\mu_c = 1$. It is known that in perfect conducting materials, ϵ and μ are interchangeable $\epsilon \leftrightarrow \mu$. Consequently, the bag is simply an ideal phenomenon of a perfect conductor. Outside the bag, $\epsilon_c = 0$ corresponding to $\mu_c = \infty$ making the QCD vacuum a perfect *paramagnetic*. As a result, the color particles are completely confined inside the bag. Also, the QCD vacuum is a perfect dielectric medium i.e. $\epsilon_c = 0$ representing a dual superconductor. In ordinary superconductor $\mu = 0$ so its dual superconductor will correspond to $\epsilon = 0$ [154].

Figure 2.2: Diagrammatic representation of how the bag emerges from the QCD vacuum



The figure on the left represents the hadron and the vacuum states and on the right side we show the boundary on the surface of the hadron and the vacuum.

Proceeding with the general idea of a perfect conductor, the boundary conditions

$$\hat{\mathbf{n}} \times \mathbf{E} = 0 \quad \text{and} \quad \hat{\mathbf{n}} \cdot \mathbf{B} = 0, \quad (2.35)$$

must be respected. On the other hand, the boundary conditions derived from Fig. 2.2 among the color particles are

$$\hat{\mathbf{n}} \times \mathbf{B} = 0 \quad \text{and} \quad \hat{\mathbf{n}} \cdot \mathbf{E} = 0. \quad (2.36)$$

Thus, at the confining phase the electric field is uniform or completely confined [150]. The classical Maxwell's equations for electromagnetism

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{and} \quad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}, \quad (2.37)$$

can be expressed in terms of the color particles by substituting $\mathbf{E} \rightarrow -\mathbf{B}$ and $\mathbf{B} \rightarrow \mathbf{E}$. This is precisely the duality phenomenon in electrodynamics leading to dual superconductivity in the confining phase. These substitutions leave the Maxwell equations invariant. Thus, the Maxwell equations for electromagnetism works well for color particles with $\mathbf{E}$ and $\mathbf{B}$ representing the electric and magnetic *color fields*. This phenomena are used in MIT bag [20] and SLAC bag models [55] for confinement.

2.3.3 Potential Models

The potential models depend largely on the motivation of the investigator, they can be inspired by QCD theory or some experimental results. The method adopted depends on the motivation, it could be relativistic or nonrelativistic quantum mechanics approach. Some of the well-known potentials used to investigate hadron properties will be reviewed in this section.

One of the famous phenomenological models built to study *color confinement* of quarks and subsequently describe the properties of hadrons are *nonrelativistic quark models* (NRQM) [135, 155, 156]. In this models, quarks are treated as constituent particles with effective *constituent masses* M . These masses are relatively larger than the usual *current quark masses* because the quarks are assumed to have mass approximately 1/3 the nucleon mass with a nonrelativistic potential

$$V(r) = -\frac{a}{r} + br \quad \text{or} \quad V(r) = ar^2; \quad (2.38)$$

where a and b are positive constants [56, 157, 158, 159]. Since kinetic energy of the light quarks u , d and s are greater than their masses, in the nonrelativistic limit, the NRQM model is suitable for studying heavy quarks such as c , b and t . However, the limitations posed by the model is overlooked because it gives an accurate account of the static properties of the excited state of hadron spectra. But its application to lighter flavour sector is still not justifiable [155].

2.3.4 Cornell Potential

Cornell potential is inspired by lattice QCD calculations. It consists of linear potential-plus-Coulomb potential. It is one of the most suitable potentials for investigating heavy quarkonium systems and has inspired other phenomenological models. This kind of potential has contributed greatly to understanding QCD theory. The Cornell potential can be expressed as

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + Kr + V_0; \quad (2.39)$$

where α_s is the strong coupling constant, K is a positive constant whose value depends on the model, V_0 is the self-energy of the particle (it is a contribution from an infrared divergence) and r is the interparticle separation distance. This potential is particularly useful because it incorporates the asymptotic freedom of QCD theory at short distances and *color confinement* at long distances. Also, it agrees with the charmonium spectrum. At $r \rightarrow 0$ there is 'one-gluon-exchange' leading to Coulomb potential dominance. At $r \rightarrow \infty$ the potential rises linearly which is consistent with the flux-tube or the string model so the second term in Eq.(2.39) dominates. In the framework of heavy quarks systems α_s , K and the 'constituent quark masses' associated with the model can be fixed by fitting experimental data [56, 160, 161].

2.3.5 Linear Potential

Historically, the string model [162, 163, 164, 165] is a product of dual models and Regge phenomenology. Even before QCD became well-known, it was considered as one of the candidates for strong interaction. Phenomenologically, the string model gives a description of hadron dynamics so, most of the theories related to *color confinement* incorporate the string as flux of gauge fields [166]. In 4-dimensional space-time, the

quark-antiquark potential can be expressed as

$$V(r) = \sigma r \sqrt{1 - \frac{\pi}{6\sigma r^2}}; \quad (2.40)$$

where σ is the string tension between quark and antiquark pairs and r is the distance between them. From this relation one can derive the meson spectrum with dependence on the internal angular momentum. However, it fails to give an account of the partonic behaviour of the color particles observed from the deep-inelastic scattering experiments.

The current viewpoint on the string model is that, as the QCD theory is the fundamental theory governing strong interactions, the string model only gives a comprehensive description of some sector in QCD theory. Similar to the chiral Lagrangian for pions, an effective theory developed for string models will be valid within a restricted domain beyond which the theory is expected to break down [167, 168].

2.3.6 Power Potential

The power potential can take several forms, for instance the Cornell potential discussed in Sec. 2.3.4 can be derived from the power potential relation

$$V(r) = -\frac{a}{r^p} + br^n; \quad (2.41)$$

where $a > 0$, $b > 0$, $2 > p > 0$ and $n > 0$. However, this section will be dedicated to Martin potential [169, 170] which agrees with charmonium and bottomonium spectra for $\alpha \approx 0.1$ and the phenomenon of *color confinement* in QCD. The potential takes the form

$$V(r) = A + Br^\alpha; \quad (2.42)$$

where A , B and α are constants which can be determined through fit. The value of α helps to determine the masses of two systems by comparison. However, very small values of α are unsuitable and sensitive to small errors in energy differences. This potential is directly inspired by experimental facts and the quark charges are contained in A and B .

2.3.7 Logarithmic Potential

The most common known logarithmic potential for confinement is the Quigg-Rosner potential [171, 172] which can be expressed as

$$V(r) = C \ln \left(\frac{r}{r_0} \right); \quad (2.43)$$

where C and r_0 are constants whose values can be determined through experimental data. This potential is suitable for studying the quark model at mass level relations. It is a nonrelativistic potential in quantum mechanics that is independent on the radial excitations of the constituent quark masses. Even though, the linear potential model $V(r) \propto r^n$ seem to have more convincing theoretical basics, the logarithmic one cannot be overruled [173]. The best fit for this potential in heavy quark systems is $C = 0.71 \text{ GeV}$ and $r_0 = 0.89 \text{ GeV}^{-1}$.

2.3.8 Richardson Potential

This potential is expressed as

$$V(r) = \int \frac{d^3q}{(2\pi)^3} \left(-\frac{4}{3}\right) \frac{12\pi}{(33-2N_f)} \frac{e^{i\vec{q}\cdot\vec{r}}}{\left[q^2 \ln\left(\frac{q^2}{\Lambda^2} + 1\right)\right]}. \quad (2.44)$$

It is particularly significant because of its dependence on Λ (QCD scale parameter). From the above expression, we can identify

$$V(q^2) = \left(-\frac{4}{3}\right) \frac{12\pi}{(33-2N_f)} \frac{1}{\left[q^2 \ln\left(\frac{q^2}{\Lambda^2} + 1\right)\right]}. \quad (2.45)$$

The Cornell potential can be obtained from the above expression by taking the limits $\vec{q} \rightarrow \infty$ and $\vec{q} \rightarrow 0$, expressed for the UV and the IR regimes respectively. For $q^2 \rightarrow \infty$

$$V(q^2) \rightarrow \left(-\frac{4}{3}\right) \frac{12\pi}{(33-2N_f)} \frac{1}{\left[q^2 \ln\left(\frac{q^2}{\Lambda^2}\right)\right]} = -\frac{4}{3} \frac{\alpha_s(q^2)}{q^2} \quad \text{for } q^2 \gg \Lambda^2; \quad (2.46)$$

here, $\alpha_s(q^2)$ is the strong coupling constant which can be obtained from the leading term in Eq.(2.15). This is the Coulomb potential in the ultraviolet regime in momentum space. Now assuming that Eq.(2.45) is also valid in the infrared regime, i.e. $q \rightarrow 0$, we get

$$V(q^2) = \frac{16\pi}{(33-2N_f)} \frac{\Lambda^2}{q^4} - \frac{8\pi}{(33-2N_f)} \frac{1}{q^2} \quad \text{for } q^2 \ll \Lambda, \quad (2.47)$$

expanded up to the Coulomb term. Bearing in mind that

$$\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^4} \propto r \quad \text{and} \quad \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2} \propto \frac{1}{r}, \quad (2.48)$$

Richardson potential [174] is simply a Cornell potential in momentum space [175, 176].

2.3.9 Heavy Quark Effective Theory (HQET)

The HQET is an effective field theory where a heavy quark symmetry shows up at large quark mass, $m_Q \rightarrow \infty$, limits, the pioneers are [177]. It is used to investigate the dynamics of hadrons with a single heavy quark Q or an antiquark $\bar{Q}$. It should be noted that heavy and light quarks are classified relative to the nonperturbative QCD scale Λ_{QCD} , dynamically generated from QCD theory. The typical size of a system of light quarks is in the order Λ_{QCD}^{-1} . A color neutral meson such as $Q\bar{q}$ consisting of a heavy quark of mass $m_Q \gg \Lambda_{QCD}$ and a light quark of mass $m_q \ll \Lambda_{QCD}$, where $m_q < \Lambda_{QCD} \leq m_Q$, are treated under HQET. In *constituent quark models* [178, 179, 180] Λ_{QCD} is related to the nucleon as $\Lambda_{QCD} \simeq m_p/3 \simeq 330 \text{ MeV}$. Hence, HQET has a validity within an energy/momentum regime much smaller than m_Q . These class of heavy-light mesons have a typical size in the order of Λ_{QCD}^{-1} and a momentum of order Λ_{QCD} . The interactions of Q and q changes the momentum of Q by Λ_{QCD} and its velocity by $\Lambda_{QCD}/m_Q \ll 1$ hence, $\Delta v = \Delta p/m_Q$, which is negligible. As a result, we can infer that Q moves with a constant velocity which is equivalent to the velocity of a heavy hadron. In this framework, the quark is presumed to be at rest [181].

This theory is of importance because, an understanding of the physics of heavy quarks will enhance the investigations and the forecasts of QCD theory and the SM. Considering a quark system with a single heavy quark with velocity v interacting with light flavours ($N_f = N_l + N_h$ where $N_h = 1$), the on-shell momentum is given as $p^\mu = m_Q v^\mu$ and an off-shell momentum $p^\mu = m_Q v^\mu + k^\mu$, where p^μ is the flavour 4-momentum, m_Q is the on-shell mass and k^μ is the so-called residual momentum but k^μ is much smaller than m_Q . For a hadron with a heavy quark, k^μ is of order Λ_{QCD} . The well-known Dirac propagator can be simplified for this system as

$$i \frac{\not{p} + m_Q}{P^2 - m_Q^2 + i\varepsilon} = i \frac{m_Q \not{v} + m_Q + \not{k}}{2m_Q v \cdot k + k^2 + i\varepsilon} \rightarrow i \frac{1 + \not{v}}{2v \cdot k + i\varepsilon}, \quad (2.49)$$

in a reference frame where $v^2 = 1$ and $k^2 = 0$ (off-shell) in the limit of heavy quarks, $m_Q \rightarrow \infty$. Therefore, there is a velocity projection operator

$$\frac{1 + \not{v}}{2}, \quad (2.50)$$

in the rest frame, it becomes

$$\frac{1 + \gamma^0}{2}, \quad (2.51)$$

which is a projection onto the particle components. The corresponding quark wave functions are

$$Q(x) = e^{-im_Q v \cdot x} [Q_v(x) + \mathcal{Q}_v(x)], \quad (2.52)$$

with

$$Q_v(x) = e^{im_Q v \cdot x} \frac{1 + \not{v}}{2} Q(x) \quad \text{and} \quad \mathcal{Q}_v(x) = e^{im_Q v \cdot x} \frac{1 - \not{v}}{2} Q(x); \quad (2.53)$$

where $Q_v(x)$ is a velocity dependent quark field and $Q(x)$ is the initial quark field at tree-level. It is obvious at this point that the mass of the heavy quark is absolutely immaterial in the limit $m_Q \rightarrow \infty$, hence all the heavy particles interact in a similar manner in the system. This is referred to as the *heavy quark symmetry*. Therefore, the heavy quark acts identical to a static external field that transform as color triplet. In the $m_Q \rightarrow \infty$ ceiling, the static heavy quark solely interact with gluons through *chromoelectric* charge, since quark-quark interactions are not possible. Because strong interactions are only possible through gluon exchange. This process is spin independent. However, spin dependant interactions are possible by *chromomagnetic* moment of the quarks and are in the order of $1/m_Q$ (insignificant). From the afore discussion, heavy hadrons are independent of spin and mass of the heavy color source. This is a consequence of an approximate internal symmetries of the QCD Lagrangian, Eq.(2.3) [181].

The HQET is one of the approaches for analytically turning the statement for ‘large quark mass’ m_Q into systematic perturbation theory by expanding Λ_{QCD}/m_Q . This is equivalent to perturbing the strong coupling constant g_s in QCD theory to all its orders. This HQET is applicable to different kinds of systems, for instance, *the atom*. The heavy quark analogy can be used to study the nucleus and the light degrees of freedom of the electron. The difference between the atomic and the hadronic systems is that; the configuration of the light degrees of freedom are perturbative in nature so, light degrees of freedom cannot be determined due to the nonperturbative nature of the strong interaction [182]. Thus, the HQET is formulated to possibly keep away from the detail description of the configuration of light degrees of freedom. This theory is suitably applied to light-heavy color singlet hadrons as described above. The HQET has been successful in theoretically investigating leptonic and semi-leptonic decay of heavy mesons. Due to its numerous successes and simplicity, it has been accepted as a favourable theory for describing light-heavy hadrons, for more details see Refs.[183].

2.4 Holographic QCD

2.4.1 Light-Front Holography

Light-Front Holographic QCD [184] started with the establishment of linkage between Dirac's light-front dynamics in $1+3$ -dimensional space time to Einstein's gravity in 5-dimensional Anti-de Sitter space-time AdS_5 . This connection is suitable for describing relativistic bound state structures of nonperturbative QCD in quantum field theory due to the frame independent nature of LF. An AdS space is known to have a maximal symmetry with negative curvature. The connection is as a consequence of AdS/CFT(Conformal Field Theory) correspondence, a theory established to be scale independent [8]. The term 'correspondence' is used because we can map a weakly interacting gravity related theory in $d+1$ -dimensional AdS space on to d -dimensional AdS space to any strongly interacting conformal field theory (such as Yang Mills theory) in d -dimensions, giving rise to the name 'gauge/gravity duality'. This mapping generates a relation between the 5-dimensional variable (z) of AdS space and the invariant impact light-front variable (ζ) in physical space-time. The main significance is to use the light-front time $x^+ = x^0 + x^3$ ('front-form') to quantize a theory instead of the standard time x^0 ('instant-form'). Dirac showed that the former form of quantization has sufficient number of kinematic generators of the Lorentz group and boost operators than the latter.

In conjugate space, the dynamic impact variable ζ measures the separation distance of the partons in the hadrons at equal light-front time. Therefore, when the key dynamical impact variable is correctly identified, we can reduce the theory to semi-classical approximation. In that case, the properties of the strong interaction dynamics are contained in some effective potential $U(\zeta)$. It is striking that in the semi-classical approximation, the light-front Hamiltonian has a structure which exactly agrees with the eigenvalue equations in the 5-dimensional AdS space. This permits us to relate the AdS wave function $\Phi(z)$ to the internal constituent structure of the hadron. This characteristic yields a relation between the coordinate z in the AdS space with the impact LF variable ζ hence, giving an intuitive understanding of the holographic variable z in light-front QCD [159, 185].

2.4.2 AdS/QCD Correspondence

In this subsection we will discuss the AdS/QCD strong running coupling ($\alpha_s(Q^2)$) and its associated β -function in 5-dimensional Anti-de Sitter space-time (AdS_5) using CFT. Since QCD is neither conformal nor supersymmetric even though it shows minimum conformal behaviour at some energy level. To achieve confinement as in QCD, we need to break the conformal symmetry of the AdS_5 space. This can be approached in several forms, we will review some of them below. The general procedure governing AdS/CFT approach to conformal gauge theories is the isomorphism of the Poincaré group and conformal transformation $\text{SO}(4, 2)$ to the group of isometries of AdS_5 space, the group then transform into AdS_5 metric

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2); \quad (2.54)$$

where R is the AdS radius. The metric is invariant under scale changes in the fifth dimensional variable $z \rightarrow \lambda z$ and $x^\mu \rightarrow \lambda x^\mu$. The z coordinate acts like a scaling variable in the Minkowski space. Different values of z will correspond to different measure of energy for which the hadron can be studied [186]. Some known functions used for distorting the AdS_5 space are:

- dilaton profiles $e^{\pm\phi(z)}$ [159, 187] or
- a factor $e^{A(z)}$.

Each specific distortion of the AdS_5 space, we achieve confinement uniquely and we can also quantize the normalizable modes as well [163]. Various studies have been done using Light Front Holographic mapping of effective classical gravity in AdS_5 space modified by a positive-sign dilaton background to produce confinement as a function of a space-like momentum Q^2 . This induces analytically, *color confinement* for nonperturbation AdS/QCD running coupling, $\alpha^{AdS}(Q^2)$, $Q^2 = -\mathbf{q}_\perp^2$ where $\mathbf{q}$ is the four-vector momentum, but $Q^2 > 0$ for a space-like momentum.

QCD theory shows a *maximal conformal* behavior in the *infrared* (IR) regime, characterized by hadronic degrees of freedom, and in *ultraviolet* (UV) regime, characterized by quarks and gluons constituent degrees of freedom. This analogy is better appreciated when one calculates the β -function and analyse it in both regimes i.e. $\beta(Q^2 \rightarrow 0) = \beta(Q^2 \rightarrow \infty) = 0$. This has been proven to be true for both pQCD and nonperturbative QCD calculations [188, 189]. These properties permit the use of equivalent

computations in a higher dimensional curved space-time where the dual theory is weakly coupled and can be described by gravity. Due to this correspondence, we can associate the fifth dimensional coordinate z of AdS_5 space with the light-front coordinate ζ [190]. It is important to note that the running coupling $\alpha_{pQCD}(Q^2)$ for perturbation QCD (pQCD) is limited to short distances or large Q^2 regimes. In a similar sense, the AdS/QCD is based on long distance interactions and small momentum exchanges between quarks and gluons [186].

To analytically obtain this, we note that the AdS_5 action looks like the general relativity action $S_{GR} \propto \int d^4x \sqrt{|g|} R/G_N$, where R is the Ricci scalar and G_N is the Newton's constant. In AdS/QCD we replace Ricci scalar with the gauge field as $\sqrt{R} \rightarrow F^{\mu\nu}$, $\sqrt{G_N}$ replaced by the gauge coupling g_5 and the metric determinant becomes $\sqrt{|g|} \rightarrow \sqrt{|g_{AdS}|} e^{\kappa^2 z^2}$ which includes the dilaton profile $e^{\kappa^2 z^2}$ to distort the AdS_5 space. Consequently, the AdS_5 action becomes

$$S_{AdS} = -\frac{1}{4} \int d^4x dz \sqrt{|g_{AdS}|} e^{\kappa^2 z^2} \frac{1}{g_5^2} F^{\mu\nu} F_{\mu\nu}. \quad (2.55)$$

The AdS_5 metric has a determinant $(R/z)^5$, and the square of the coupling g_5 has a length dimension hence, we can relate the prefactor

$$g_5^{-2}(z) = g_5^{-2} e^{\kappa^2 z^2}; \quad (2.56)$$

which restores the conformal symmetry and brings about confinement. Thus, in the AdS_5 action the effective coupling depends on the length scale z . Hence, the coupling $g_5(z)$ carries the non-conformal color confinement dynamics exhibited by QCD theory.

2.4.3 AdS/QCD Running Coupling, $\alpha_s(Q^2)$

Generally, the strong coupling constant $\alpha_s \equiv g^2/4\pi$ acquires its effective Q^2 -dependence at short-distance quantum effects in QCD theory whilst the initial coupling constant on the AdS_5 space $\alpha_{AdS} \equiv g_{AdS}^2/4\pi$ is redefined to absorb the long distance confinement forces [191]. Accordingly, the length scale z is associated with the invariant LF impact variable ζ thus, we can relate $g_{AdS}(z) \rightarrow g_s(\zeta)$ consequently,

$$\alpha_s^{AdS} = \frac{g_{YM}^2(\zeta)}{4\pi} \propto e^{-\kappa^2 \zeta^2}. \quad (2.57)$$

Indeed, the real coupling measured on Q^2 scale is the light-front (LF) transverse two-dimensional Fourier transform of the coupling $\alpha_s^{AdS}(\zeta)$. Therefore,

$$\alpha_s^{AdS}(Q^2) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta); \quad (2.58)$$

where J_0 is a Bessel function, in an Hankel transform, which is easily evaluated after substituting the exponential equivalent of $\alpha_s^{AdS}(\zeta)$, yielding

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}; \quad (2.59)$$

where $\alpha_s^{AdS}(0)$ is a constant. The choice of the positive dilaton profile was necessary because the integral diverges at $\zeta \rightarrow \infty$ for a negative dilaton profile. The coupling $\alpha_s^{AdS}(Q^2)$ is classified with the physical QCD running coupling in the nonperturbative regime, it determines the large distance space-time dependence of the running coupling in four-dimensions. Hence, $\alpha_s^{AdS}(Q^2)$ can be normalized to $\alpha_s(g_1)$ scheme by exploring the similarities in the behavior of their β -functions; it vanishes in the infrared domain, rise to higher negative intermediately and fall-of to zero in the UV domain. We can take advantage of this similar futures to fix $\alpha_s^{AdS}(Q^2 = 0) = \pi$, to extend the AdS₅ coupling to the UV domain consistent with pQCD.

2.4.4 Holographic β -function

The nonperturbative β -function obtained from the AdS₅ space modified by positive dilaton background using the renormalization group theory can be obtained from the expression,

$$\beta^{AdS}(Q^2) = \frac{d}{d \ln(Q^2)} \alpha_s^{AdS}(Q^2) = -\frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}. \quad (2.60)$$

We can interpret that $Q_0 \simeq 2\kappa$ is the transitions from the nonperturbative IR domain to the perturbative UV domain. From this results, we can infer that at $Q < 2\kappa$ is nonperturbation regime while $Q > 2\kappa$ is the perturbation regime [191]. We also observe that the general requirements

$$\beta(Q \rightarrow 0) = \beta(Q \rightarrow \infty) = 0, \quad (2.61)$$

$$\beta(Q) < 0 \quad \text{for} \quad Q > 0, \quad (2.62)$$

$$\begin{aligned} \frac{d\beta}{dQ} &= -\frac{\pi Q}{2\kappa^2} \left[1 - \frac{Q^2}{4\kappa^2} \right] e^{-Q^2/4\kappa^2} \\ \frac{d\beta}{dQ} \Big|_{Q=Q_0} &= 0, \end{aligned} \quad (2.63)$$

$$\frac{d\beta}{dQ} < 0 \quad \text{for} \quad Q < Q_0 \quad \text{and} \quad \frac{d\beta}{dQ} > 0, \quad \text{for} \quad Q > Q_0, \quad (2.64)$$

are all satisfied. Equation (2.61) shows the conformal nature of the QCD in both the IR and the UV regime.

2.5 Lattice QCD

Lattice QCD involves the discretization of four-dimensional space-time Euclidean lattice to solve QCD numerically. It is based on formulation of path integrals of the associated action on a discrete lattice. In this regard, the quarks and the antiquarks live on the discrete points of the lattice, whilst the gluons span the links between the points. The size of the lattice depends on the problem to be solved and the computing resource available. Different kinds of actions are available for use, but a choice is made based on its merits and demerits. Again, when the grid and the action to be used are settled on, a damping factor β needs to be chosen. For instance, a large β in a particular action can lead to a smaller spacing, meanwhile same β in a different action can lead to a different result. Most lattice studies are based on quenched approximation without a full consideration of gluon fluctuations in $q\bar{q}$ pairs. However, due to emergence of high power computers and evolving new calculation techniques, things are beginning to change.

These days lattice QCD has matured and gained some status of precision in physics. It has passed several tests in low energy strong interactions leading to the strong coupling constant, which is one of the fundamental parameters useful for the quest of precision at the LHC. The starting point of lattice QCD is the construction of nonperturbative quantum field theory. Accordingly, the path integral of the theory in Euclidean spacetime is usually represented as

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu e^{-S[\bar{\psi}\psi A_\mu]}, \quad (2.65)$$

with

$$S[\bar{\psi}\psi A_\mu] = \int d^4x \left\{ \frac{1}{2g^2} \text{Tr}\{F_{\mu\nu}F_{\mu\nu}\} + \sum_{f=1}^{N_f} \bar{\psi}_f (D_\mu \gamma^\mu + m_f) \psi \right\}. \quad (2.66)$$

Lattice Field Theory deals with an accurate description of path integral by discretizing the spacetime into hyper-cubic lattice with spacing a . Matter fields at the lattice sites are represented as $x_\mu = an_\mu$, $n_\mu = 1, \dots, L_\mu/a$, where L_μ is the physical size in μ

direction. After the discretization the integral is over many degrees of freedom which can be limited by definitions. The lattice spacing a , also serves as the UV cut off of the theory. Comprehensive reviews of the subject can be found in [192, 193, 194, 195] and references therein.

2.6 Glueballs and Glueball Masses

2.6.1 Theoretical Contributions

QCD is the fundamental theory for strong interactions, but it is difficult to solve in the low energy regime, where particles in today's universe can be described. However, this challenge is gradually changing due to the advances in lattice QCD and the emergence of faster computers. Particles that are made up of $q\bar{q}$ systems can be observed in the meson sector and can be described in the framework of the quark model. However, phenomenological models and lattice calculations points to the existence of a particle in which the gluons can contribute to its quantum number. This brings about pure-gluon objects referred to as the glueballs also, $q\bar{q}$ states with pure-gluon mixing states known as hybrid mesons. Particles formed in this manner are likely to have quantum numbers which are forbidden in $q\bar{q}$ systems—exotic quantum numbers. In recent years, several experimental data have been collected and other ongoing experiments are expected to provide a precise data to describe this phenomenon [196, 197, 198, 199, 200].

Before we discuss the gluonic excitations into detail, we will throw a light on the quark model for mesons. Mesons are composed of $q\bar{q}$ system with a negative intrinsic parity due to the presence of particle and an antiparticle. The parity takes the form $P = -(-1)^L$, where L is the orbital angular momentum. Since quarks are spin(S)-1/2 particles, mesons have a total spin of either $S = 0$ or $S = 1$, with a total angular momentum $J = L + S$, the sum is made in conformity with the rules of summing angular momentum. Besides, the system has a charge conjugation given as $C = (-1)^{(L+S)}$. In a system of light quarks, there is an additional quantum number known as the *isospin quantum number* (I). In this case, each quark is assigned an isospin-1/2, conversionally u-quark has $I = +1/2$ and d-quark, $I = -1/2$. Thus, a meson formed out of these particles possess' a total isospin of either $I = 0$ or $I = 1$. Consequently, a system involving a strange quark leads to a total $I = 1/2$, if both quarks are strange quarks then $I = 0$. This

brings to light a new conserve quantum number, G-parity as $G = (-1)^{(L+S+I)}$. Adopting these quantum numbers we can develop J^{PC} 's for mesons, we get an allowed quantum numbers

$$0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{--}, 2^{--}, 2^{-+}, 2^{++}, 3^{--}, 3^{+-}, 3^{--}, \dots, \quad (2.67)$$

following these quantum numbers, we observe that there is an existing sequence of J^{PC} 's which are not consistent with a simple $q\bar{q}$ systems, i.e,

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots. \quad (2.68)$$

These are *exotic quantum numbers*, if observed will lead to other particles beyond just $q\bar{q}$ system of the quark model.

One of the pioneering models for determining glueballs is the Bag Model [20]. In this model, boundary conditions are imposed to confine the gluons inside the Bag to form either color neutral or colorless bound states [201]. Under this model the gluon fields come in two forms i.e., transverse electric TE or transverse magnetic TM with parities $(-1)^{J+1}$ and $(-1)^J$ respectively. Under this scenario, the gluons have to populate the Bag to form color singlet states. Consequently, two and three gluon-gluon states are expected to form. Since glueballs do not contain quarks, they are expected to couple with any quark flavour in the same manner. The glueballs undergo a decay process with decay constant/amplitude γ^2 in a simple SU(3) calculation from the expression

$$\Gamma = \gamma^2 \cdot f_L(q) \cdot q; \quad (2.69)$$

where Γ is the decay rate, $f_L(q)$ is the form factor that depends on the angular momentum L and q is the break-up momentum of the meson into a pair of daughter mesons. One of the most significant characteristics of decaying glueballs into a pair of pseudoscalar mesons is the vanishing branching ration of the process $G \rightarrow \eta\eta'$. That notwithstanding, a missing $\eta\eta'$ states can also be as a results of interference of different components of $q\bar{q}$ states [20, 201, 202, 203].

Table 2.3: Glueball masses determined from the Bag Model

Glueballs and their properties		
Gluons	J^{PC} quantum number	Mass (GeV/ c^2)
$(\text{TE})^2$	$0^{++}, 2^{++}$	0.96
$(\text{TE})(\text{TM})^2$	$0^{-+}, 1^{-+}, 2^{-+}$	1.29
$(\text{TE})^3$	$0^{+-}, 1^{++}, 2^{+-}, 3^{++}$	1.46
$(\text{TM})^2$	$0^{++}, 2^{++}$	1.56

Detailed description can be found in Ref.[201].

Table 2.4: Decay amplitudes for glueballs normalized at $\eta\eta$

Decay of glueballs					
Decay	$\pi\pi$	$K\bar{K}$	$\eta\eta$	$\eta\eta'$	$\eta'\eta'$
γ^2	3	4	1	0	1

Furthermore, Isgur and Paton [204] were first to calculate the glueball masses using flux-tube model, where they treated the glueball as a closed tube. They also obtained the mass of the lightest glueball mass with quantum number $J^{PC} = 0^{++}$ to be 1.52 GeV/ c^2 . Further calculations lead to the prediction of three lightest glueball masses [205]. This prediction was found to be consistent with the results obtained from the lattice calculations [206] (see Tab.2.5). These opened the way for several flux-tube calculations to determine scalar, tensor and pseudoscalar glueball masses. Consequently, predictions have been made through the flux-tube model that the scalar and pseudoscalar glueballs should have a degenerate mass [207].

Table 2.5: Flux-tube prediction of lowest lying glueball masses

Glueball masses		
$J^{PC} = 0^{++}(\text{GeV}/c^2)$	$J^{PC} = 2^{++}(\text{GeV}/c^2)$	$J^{PC} = 0^{-+}(\text{GeV}/c^2)$
1.52 [204]	2.84[204]	2.79[204]
1.68[205]	2.69 [205]	2.57[205]
1.60[208]		

Another way of calculating glueball masses is through the QCD sum rules. This approach makes use of two-point correlator of the suitable field operators from QCD and generators of sum rule by juxtaposing a dispersion relation for the correlator to an Operator Product Expansion. The pioneer for calculating glueballs through this approach is Novikov [209] and has since then attracted some attention. QCD sum rule is particularly suitable for exploring QCD in the intermediate energy level. Most of these calculations yields lightest scalar glueball mass within 0.3 to 0.6 GeV/ c^2 [210]. Recent results [211] exploring all the known scalar mesons points to a scalar meson mass of about 1 GeV/ c^2 and a scalar glueball mass of 1.5 ± 0.2 GeV/ c^2 . In this framework, the heaviest glueball mass seem to emerge from the "two Laplace subtracted sum rule", whilst the unsubtracted sum rules produces the lower glueball masses [212].

Additionally, Swanson and Szczepaniak calculated the glueball spectrume using Hamiltonian QCD method in Coulomb gauge [213] by developing a quasiparticle gluon basis. Their outcome was in good agreement with the lattice calculations. They obtained a lightest scalar glueball mass to be 1.98 GeV/ c^2 , pseudoscalar glueball mass of 2.22 GeV/ c^2 and a tensor glueball mass of 2.42 GeV/ c^2 .

There are several ways of calculating scalar glueballs [214] from lattice QCD with some few challenges associated with each approach adopted. In some cases, there is a singularity which is not associated with QCD that makes the scalar glueballs artificially small. This phenomenon is common when Wilson fermions are employed with large lattice spacing. Other related mechanisms are less sensitive to this behaviour, small lattice spacing also help in eliminating this effect. Most of the pioneering lattice calculations of the glueball spectrum were done out of quenched approximations on comparatively small lattices [215]. These calculations initially pointed to lightest glueball with mass of about

$1.5 \text{ GeV}/c^2$. With the emergence of new computational methods and improved actions, large lattices are being considered and the spectrum of the states have began showing some improvement [352]. Extrapolating to the continuum limit, the observed lightest glueball states are; scalar ($J^{PC} = 0^{++}$) with mass $1.55 \pm 0.05 \text{ GeV}/c^2$, tensor ($J^{PC} = 2^{++}$) with mass $2.27 \pm 0.1 \text{ GeV}/c^2$, and pseudoscalar ($J^{PC} = 0^{-+}$) with almost the same mass. The first exotic state for non- $q\bar{q}$ quantum number was also obtained to be above $3 \text{ GeV}/c^2$. When both small and large parameters were used the resulting glueball mass was obtained to be $1.625 \pm 0.094 \text{ GeV}/c^2$ [399, 216]. These authors also identified for the first time that glueballs do not necessarily decay in flavour-blind fashion. This violations were further investigated by Refs.[217, 253]. This phenomenon was accomplished by introducing a parameter r , in the matrix that mixes quarkonium with glueballs. For flavour blind decays $r \simeq 1$, on a lattice [399, 216] $r = 1.2 \pm 0.07$, while fitting to a data [217] produces $r = 1.2 \pm 0.3$.

Table 2.6: Glueball spectrum obtained from lattice calculations.

Glueball masses	
J^{PC}	$M_G(\text{GeV}/c^2)$
0^{++}	1.710(0.050)(0.080)
2^{++}	2.390(0.030)(0.120)
0^{-+}	2.560(0.035)(0.120)
1^{+-}	2.980(0.030)(0.140)
2^{-+}	3.040(0.040)(0.150)
3^{+-}	3.600(0.040)(0.170)
3^{++}	3.670(0.050)(0.180)
1^{--}	3.830(0.040)(0.190)
2^{--}	4.010(0.045)(0.200)
3^{--}	4.200(0.045)(0.200)
2^{+-}	4.230(0.050)(0.200)
0^{+-}	4.780(0.060)(0.230)

The table contains the glueball spectrum in physical units. There are two different error margins calculated for each value, the one at the immediate right of the value is the combined uncertainty in $r_0 M_G$ and the other at the far right is the uncertainty in $r_0^{-1} = 410(20) \text{ MeV}$. The data for this results is contained in [29, 218].

2.6.2 Experimental Signature

Meson spectroscopy derive its outcome from various experimental techniques. Glueball production/formation have generally been grounded on glue-rich environments [219]. Chief among the various approaches are the $p\bar{p}$ annihilation, central pp collisions through double Pomeron exchange or radiative decay of quarkonia, in this case, one of the three gluons arising from the process is substituted with a photon while the two gluons self-interact to form bound states, for instance the radiative decay of $J/\psi \rightarrow \gamma G$ [220, 221]. We will highlight major experiments dedicated for investigating meson resonances on the search for glueballs.

In $p\bar{p}$ annihilation, glueballs are expected to form when $q\bar{q}$ annihilate leading to

the formation of gluons. This process will results in the formation of glueballs without recoil particle, this prohibits the formation of exotic quantum numbers, so the properties of glueball candidates can be determined from the initial states. On the contrary, production of heavier resonances recoiling against another meson is usually expected, however, bound states of gluons forming glueballs are also likely. Additionally, the commonly observed final states made up of u and d -quarks can be produced through quark arrangements without glueballs formed at the intermediate states. It has also been observed in $p\bar{p}$ annihilations that meson mass of about $1.7\text{ GeV}/c^2$ is produced at rest. Data collected, reviews and on ongoing experiments intended to collect more exact data on this subject can be found in Refs.[222, 223, 224, 225, 226, 227, 196, 228, 229, 230].

Radiative decay of quarkonia and e^+e^- annihilation experiments are some of the most reliable mechanisms used in search for glueballs. Information regarding glueballs are mostly concentrated in J/ψ decay after a photon is emitted from $c\bar{c}$ annihilation leading to C-even gg states. As a result, it is expected to have strong coupling to low-lying glueballs. Also, radiative decays of $\Upsilon(1S)$ are glue rich processes and are associated with a list of two-body branching ratios, some works have been done by CLEO collaboration [231, 232] in this regard. Besides, J/ψ decay can be used to investigate flavour content of mesons in a so-called *flavour-tagging* process. Consequently, a flavour filter was proposed for glueballs with quantum numbers 0^{++} , 0^{-+} and 2^{++} states in a decay process $J/\psi \rightarrow \gamma [\gamma V]$ with $V = \rho, \phi$ [233]. Again, the decay of $c\bar{c}$ can be investigated through e^+e^- collision via the emission of a virtual photon in the process

$$e^+e^- \rightarrow \gamma^* \rightarrow c\bar{c}. \quad (2.70)$$

Other processes have been investigated in the BES experiment at e^+e^- collider BEPC at IHEP, Beijing [234]. Radiative decay of $c\bar{c}$ and $\Upsilon(b\bar{b})$ was successively investigated at CLEO [231, 232, 235, 236, 237, 238], CLEO-II, and CLEO-III [239].

We now look at how glueballs are produced through *central production*. Unlike the formation experiments discussed in the previous paragraphs, the total energy in production experiments are divided between the recoiling particle(s) and a multi-meson final state. Creation of new particle is dependant on the center-of-mass energy of the experimental set-up. It has been proposed that glueballs can be produced through *central production* [240],

$$\text{hadron}_{\text{beam}} p \rightarrow \text{hadron}_f X p_s; \quad (2.71)$$

here, the final state carries some fractions of the initial state hadron momenta. To achieve this results through a (proton) fixed target experiment, a slow proton and a fast hadron can be seen at the final state. The outcome of the *central production* points to an enhanced glueball candidate as compared to the conventional $q\bar{q}$ meson, this triggered the idea of glueball filter [241]. It has been argued that glueball filter may just be a momentum filter which reduces the angular momentum and thus improves the scalar production [242]. Many experiments have been set-up over the years to investigate this subject, particularly at CERN [243] and also WA76 [244] and others [218, 245, 246, 247].

Other experimental processes including Two-Photon-Fusion through e^+e^- collider are also helpful. Aside the direct signs of the presence of glueball states, its absence is equally worth investigating to obtain the clear picture. Generally, glueballs are not expected in two-photon fusion because it has no valence charge to couple to photons. That notwithstanding, a small branching ratio for $X \rightarrow \gamma\gamma$ do not give a conclusive information as to whether X is a glueball or not, because of an interference effect. For instance, the branching ratio for the resonance, $f_0 \rightarrow \gamma\gamma$ gives rise to $f_0(1710) : f_0(1500) : f_0(1370) \approx 3 : 1 : 12$ [221]. The collision of two-photons are well investigated in an inelastic Bhabha scattering in e^+e^- collider through $e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-X$ [248]. Each of the photon is massless but the combination of the two can form a massive state X . Some results for these processes have been reported by CELLO in Refs.[237, 249]. Other experiments have contributed to the meson spectroscopy but were not directly involved in the search for glueballs. For instance, the pion and kaon beams in charge exchange processes,

$$\pi^-(K^-) + \text{Proton} \rightarrow \text{neutron} + \text{meson}. \quad (2.72)$$

The LASS facility at SLAC established for strangeonium spectroscopy [250], the VERTEX spectrometer setup [251], experiment E852 [252] at Brookhaven National Laboratory, Multi-Particle Spectrometer Facility (MPS) of Brookhaven's and Alternating Gradient Synchrotron (AGS) are all associated experimental setups.

2.7 Maxwell Equations

Maxwell equations are the elementary equations governing classical electrodynamics. These equations follow the postulate, basic laws of physics should not vary from one inertial frame to another through Lorentz transformations. In relativity, the electric and

magnetic fields are seen as different fields in the same entity — electromagnetic field. In view of this, the Maxwell equations in a rationalized metric system are expressed as

$$\begin{aligned}
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\
\nabla \times \mathbf{B} &= \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{j}}{\varepsilon_0}, \quad \text{and} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}.
\end{aligned} \tag{2.73}$$

In Gaussian units, they take the form

$$\begin{aligned}
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \cdot \mathbf{E} &= 4\pi\rho, \\
\nabla \times \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} + 4\pi\mathbf{j}, \quad \text{and} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}.
\end{aligned} \tag{2.74}$$

Aside these equations, some useful identities such as

$$\begin{aligned}
\mathbf{j} &= \sigma \mathbf{E}, \\
\mathbf{D} &= \varepsilon \mathbf{E}, \quad \text{and} \\
\mathbf{B} &= \mu \mathbf{H},
\end{aligned} \tag{2.75}$$

are necessary for proper understanding. In the above expressions, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic induction (mostly used interchangeably with the magnetic field), ρ is electric charge density, $\mathbf{j}$ is electric current density, ε_0 is electric permittivity in free space, $\mathbf{D}$ is electric displacement, $\mathbf{H}$ is the magnetic field, σ is conductivity, ε is dielectric constant (permittivity) and μ is the magnetic permeability. These equations reflect the coexistence of the electric and magnetic fields in nature.

The above listed four Maxwell equations [258] can be written in tensor form as

$$\partial_\mu F^{\mu\nu} = j^\nu; \tag{2.76}$$

which obey the rotation relation

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0, \tag{2.77}$$

while

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{bmatrix}, \quad (2.78)$$

with

$$j^\nu = (\rho, j_1, j_2, j_3) \quad (2.79)$$

and

$$\partial_\mu = \left[\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right]. \quad (2.80)$$

The Lagrangian density that gives rise to Eq.(2.76) is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\nu A_\nu; \quad (2.81)$$

where A_ν is the four vector potential, the time component is the electric potential Φ while the spatial component is the vector potential $\mathbf{A}$ whose curl is the magnetic field $\mathbf{B}$ i.e. $\mathbf{B} = \nabla \times \mathbf{A}$ [259, 260]. We will treat the modification of the Maxwell equations with source and without source immersed in color dielectric medium, denoted by the function $G(\phi)$ in the subsequent subsections. The behaviour of the electromagnetic field in the dielectric function will be of immersed significance to this thesis. Because the dynamics of the dielectric function will be a source of confining electric field and QCD-like vacuum.

2.7.1 Maxwell's Lagrangian Modified by $G(\phi)$

Using the Lagrangian Eq.(2.81) with its equations of motion Eq.(2.76), i.e. Maxwell's Lagrangian with source. The equations of motion of the electromagnetic field create spherical symmetric solutions similar to the well-known point charge scenario associated with Coulomb's field [261]. Therefore, for electromagnetic field immersed in a dielectric medium $G(\phi)$, where $\phi(r)$ is the scalar field representing the dynamics of the medium, we can construct a Lagrangian density

$$\mathcal{L} = -\frac{1}{4}G(\phi)F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu, \quad (2.82)$$

with its equations of motion given as

$$\partial_\mu [G(\phi)F^{\mu\nu}] = j^\nu, \quad (2.83)$$

these equations have exact solutions. Going forward, we fix the indices to be $\mu = j = 1, 2, 3$ and $\nu = 0$. These choices are made to ensure that there is *chromoelectric flux* confinement and to eliminate *chromomagnetic flux* in the rest frame of the associated point particles. As a result, the equations of motion Eq.(2.83) can be reduced to

$$\nabla.[G(\phi)\mathbf{E}] = j^0 = \rho. \quad (2.84)$$

By this equation, we have eliminated the effect of magnetic field contribution leaving only the electric field contribution which is sufficient to achieve the objective of this thesis. From Eq.(2.84), $\mathbf{E}$ is the electric field coupled to the dielectric function $G(\phi)$. Rewriting this equation in spherical coordinates, making use of the assumption that $E(r)$ and $\phi(r)$ depends only on the radial distance r , we can write

$$\nabla.[G(\phi)\mathbf{E}] = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 G(\phi) E_r) = \rho. \quad (2.85)$$

Integrating this differential equation yields,

$$\begin{aligned} [r^2 G(\phi)\mathbf{E}] &= \frac{\rho}{\varepsilon_0} \int_0^R r^2 dr \\ E_r &= \frac{q}{4\pi\varepsilon_0 r^2 G(\phi)}; \end{aligned} \quad (2.86)$$

where $q = \frac{4}{3}\pi R^3 \rho$, $E = |\mathbf{E}| = E_r$, R is the radius of the hadron and r is the intra-particle separation inside the hadron. We observe that the dielectric function modifies the magnitude of the electric field function thus, inducing the characteristics required to analyse *color confinement* and its associated properties. To successfully investigate *color confinement*, the dielectric function must satisfy the conditions

$$G(\phi(r)) \rightarrow \begin{cases} 1 & \text{for } r \rightarrow 0 \\ 0 & \text{for } r \rightarrow r_* \end{cases}, \quad (2.87)$$

where r_* is the separation distance between the particles for which confinement begins. These conditions are similar to the behaviour of the color dielectric constant ε_c obtained from the bag model Eq.(2.34).

2.7.2 Maxwell's Lagrangian Modified by $G(\phi)$ without Source

Continuing the review of the theory of electromagnetism in *color dielectric* medium using Maxwell's equation without source, i.e. using Eq.(2.81) with $j^\nu = 0$. Its equations

of motion are

$$\partial_\mu F^{\mu\nu} = 0. \quad (2.88)$$

It is worth mentioning that though the Lagrangian is with no source, its equations of motion still admit solutions in spherical symmetry [262].

Considering the gauge field in dielectric medium, $G(\phi)$, the Lagrangian can be rewritten as

$$\mathcal{L} = -\frac{1}{4}G(\phi)F_{\mu\nu}F^{\mu\nu}. \quad (2.89)$$

With equations of motion

$$\partial_\mu[G(\phi)F^{\mu\nu}] = 0. \quad (2.90)$$

Using similar indices as defined in Sec. 2.7.1 we can express

$$\nabla.[G(\phi)\mathbf{E}] = 0. \quad (2.91)$$

As stated above, the magnetic field is of no interest to us so, we can concentrate on the electric field confinement. Utilizing Eq.(2.91), we can determine the solution of the electric field $\mathbf{E}$ in the dielectric medium $G(\phi)$ to be

$$\nabla.[G(\phi)\mathbf{E}] = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 G(\phi) E_r) = 0, \quad (2.92)$$

and

$$E_r = \frac{\lambda}{r^2 G(\phi)}. \quad (2.93)$$

Here, λ is the integration constant which can be related to $\lambda = q/4\pi\epsilon_0$. As a result, the electric field solution $\mathbf{E}$ in the dielectric medium $G(\phi)$ can be represented as

$$E = \frac{q}{4\pi\epsilon_0 r^2 G(\phi)}. \quad (2.94)$$

Consequently, the dielectric media changes the strength of the $\mathbf{E}$ as a function of ϕ similar to Eq.(2.86).

2.8 Summary

In this chapter we reviewed briefly the history of quark model, its classification and the classical QCD Lagrangian citing the pioneers behind this novelty. We looked at some successful phenomenological models that have made impact in understanding hadron

spectrum or *color confinement* phenomena. Again, considering the recent advances made by holographic models in explaining *color confinement* and hadron spectrum, we briefly discussed light-front holography and AdS/QCD. The Maxwell equations reviewed here are intended to particularly show how the electric field behaves in a dielectric medium and to prepare the grounds for its applications in the main work. This chapter is to set the pace for the original contribution of the thesis and to bring non expert readers up to speed and prepare them to appreciate the main work.

From the classical QCD Lagrangian, we find that the strong running coupling $\alpha_s(\mu)$ decreases with increasing momentum, representing the *asymptotic free* nature of the particles governed by the theory. Moreover, the QCD β -function derived from the theory is naturally negative, showing the uniqueness of the strong interaction phenomena. The uniqueness of this theory leads to *color confinement* which is not yet well understood by the community of physicists. The self-interacting nature of the associated gauge bosons and their *color charges* makes it distinct from the QED theory. However, both theories (QCD and QED) are governed by perturbation theory. Moreover, QCD shows an interesting feature in the *chiral limit* where the effective mass of the theory is zero. This limit is significant in understanding the creation of asymptotically free quarks, pions, deconfinement and QGP phase transitions, a phenomenon necessary for exploring the dynamics of the *early universe*.

The main objective of this thesis is to develop phenomenological QCD models therefore, there is a need to throw some light on some successful ones. We looked at the *bag model* which is the pioneer model built to provide explanation to *color confinement*. It is based on the idea of a bubble in the QCD vacuum where the bubble is presumed to contain the free quarks and gluons. Consequently, the bubble is referred to as the bag or the hadron. The bag is seen as a *perfect conductor* and the vacuum as *paramagnetic medium*. So, there is a boundary between the bag and the vacuum which does not allow the penetration of color particles in and out of the bag.

Furthermore, due to the prior knowledge on QCD i.e. confinement and asymptotically free nature of QCD theory affirmed by experiment and QCD lattice simulations, some potential models have been developed to offer some explanations. These potential models are based on different inspirations and motivations of the investigator. However, the Cornell potential which incorporate linear confinement and asymptotic freedom in-

spired by lattice calculations is one of the popularly adopted potentials largely accepted in the scientific community. Another popular potential is the linear potential model. It conforms with experiment and QCD lattice stimulations even though, this potential existed before QCD theory became well known. However, it is only viable for exploring the infrared regime of the theory where *color confinement* is eminent. This potential mostly comes up in developing *flux tube* models for *color confinement*. All the other potential models are derived from these two potentials in either momentum space or in logarithmic form. The HQET on the other hand, has enormous theoretical advantage and it is easy to deal with.

Light-front holography provides a means for establishing an exact relation between the boost-invariant LF wave function for analysing the internal structure of hadrons in physical space-time with the bound-state amplitudes in AdS space. Therefore, the five dimensional AdS variable z can be reduced to LF impact variable ζ in relativistic domain and onto the radial variable r in nonrelativistic domain thus, $z \rightarrow \zeta \rightarrow r$. Again, holographic principles are based on the AdS/CFT correspondence [8, 9, 10]. The correspondence permits the establishment of relation between $d+1$ -dimensional Anti-de Sitter space and d -dimensional conformal field theory (CFT). Based on this correspondence, we establish a relation between $SU(N)$, $\mathcal{N} = 4$, supersymmetric Yang Mills (SYM) theory —a maximally supersymmetric gauge field theory in 4-dimensional space time — and gravity. This equivalence gave rise to the strong running coupling $\alpha_s(Q^2)$ and the β -function. Other hadronic and confinement properties can also be calculated from this correspondence.

Chapter 3

Effective Theory for Tachyons Coupled with SM Particles

This and the following chapters will contain our original contributions to the phenomenon of strong interactions. In this particular chapter we will develop the relevant theories that will be used in subsequent chapters for further investigations.

3.1 Review of Tachyons and Dp Branes

3.1.1 Tachyons

Tachyons are perceived as weakly interacting superluminal particles or particles that travel faster than light. For a relativistic particle with energy $E^2 = p^2c^2 + m^2c^4$, where p is the particle momentum, m^2 is mass and c is the speed of light in vacuum, for it to have a speed greater than the speed of light in vacuum, the relativistic velocity $\beta = pc/E$ must be greater than unity, i.e. $\beta > 1$. So, for a tachyon with positive momentum, the mass must be imaginary [263]. This argument does not seem to make a strong case for the existence of tachyons, however, Quantum Field Theory (QFT) provides much convincing description of tachyon dynamics. In light of QFT, particles that travel faster than light do not exist in nature, so tachyons are seen as particles that are unstable and undergo decay. Under this picture, tachyons are associated with scalar fields, say ϕ , with a kinetic term and a potential $V(\phi)$ whose extremum is at the origin. If one carries out perturbation quantisation about $\phi = 0$ up to the quadratic term, leaving out the higher order terms, a particle-like state is expected at $V''(0)$. Under the circumstance,

this result have two different interpretations; when $V''(0) > 0$, we have a particle with real mass m_ϕ , and when $V''(0) < 0$, we have a tachyonic state with an imaginary mass m_ϕ . This phenomenon gives physical meaning to tachyons. Since tachyons have negative square mass $-m_\phi^2$ whose potential has its extremum at the origin, and hence are associated with instability, a small displacement at the origin will cause, ϕ , to grow exponentially with time towards the true minimum. Accordingly, the perturbation expansion of the potential takes the form

$$V(\phi) = -\frac{1}{2}m_\phi^2\phi^2 + c_3\phi^3 + c_4\phi^4 + \dots; \quad (3.1)$$

where c_3 and c_4 are constants. Thus, the tachyons behave like Higgs particles with negative square mass when the Higgs field, $H = 0$. So, the unstable nature of the tachyons breakdown the perturbation theory, in that case, the usual perturbation theory in QFT are not applicable. The maximum of the potential at $\phi = 0$ render it classically unstable to be expanded around that same point, because we can not guarantee that the fluctuations at this point is small. However, this comes with an inbuilt correction. One can perturb the potential about its true minimum ϕ_0 up to the quadratic term, ignoring the higher order terms. This will lead to the creation of a particle with positive square mass m_ϕ^2 at $V''(\phi_0)$, removing the tachyonic modes in the spectrum.

Additionally, in non-BPS systems with tachyons, the theory is constructed with invariant Z_2 symmetry, $\phi \rightarrow -\phi$, and when it contains brane and anti-brane (here the system must be represented with complex scalar field), it must also be invariant under the global phase transformation, $\phi \rightarrow e^{i\alpha}\phi$. As a result, there are some advantages that motivates one to work with tachyonic systems, otherwise, they can be removed by defining a new field $\eta = \phi - \phi_0$ and perturb around the new field such that

$$V(\eta) = V(\phi)|_{\phi_0} + V'(\phi)|_{\phi_0}\eta + \frac{1}{2}V''(\phi)|_{\phi_0}\eta^2. \quad (3.2)$$

Proceeding with this potential, the tachyonic modes have been removed giving the new field η a real mass. However, the Z_2 symmetry in $V(\eta)$ is not as explicit as it is in $V(\phi)$. Meanwhile, high Z_2 symmetry among tachyons leads to the phenomenon of spontaneous symmetry breaking, where the potential presents a degenerate vacua at ϕ_0 and $-\phi_0$. This is an important phenomenon in investigating elementary particles in particle physics [264].

3.1.2 Dp Branes

Recent advances in string theory helps to explain the characteristics of nonperturbative nuclear matter using Dp-branes as the background particles. Dp-branes throw light on physically relevant systems such as *black holes*, *supersymmetric gauge theories*, and a connection between Yang-Mills and *quantum gravity theories*. Even though there have been immersed progress in this direction, a consistent background-independent development of strongly interacting matter theory is still under construction. This creates challenges in directly addressing some cosmological problems as well. Supersymmetric closed strings can be quantized in five different ways, namely; IIA, IIB, I, heterotic SO(32) and heterotic $E_8 \times E_8$ superstring theories. They are separately investigated under perturbative quantum gravity. However, they interrelate through duality symmetry transformations [265, 266]. That notwithstanding, some dualities are unperturbative in form because the string coupling g_s in one theory can have an inverse relation $1/g_s$ with another. Consequently, both the superstring theory and M-theory (unification of superstring theories) consist of Dp-branes, but in different configurations. For instance, IIA/IIB superstring theories are made up of even/odd Dp-brane configurations. Adopting the appropriate transformations, a Dp-brane in one theory can be related to the other in another theory, even including their connecting strings. This process is affectionately called, 'brane democracy', hence none of the brane is more fundamental than the other.

The study will generally center around open strings with their endpoints on Dp-brane. However, we will throw light on both open and closed strings here, while we narrow down to the specific string intended for the study in the subsequent sections. Closed strings are topologically the same as circle S^1 , i.e. $[0, 2\pi]$. When quantised, they give rise to massless set of spacetime fields, such as graviton $g_{\mu\nu}$, dilaton φ , antisymmetric two-form $B_{\mu\nu}$ and an infinite set of massive fields. On the other hand, supersymmetric closed strings consist of massless fields identified as; graviton supermultiplet such as Ramond-Ramond p -form fields $A_{\mu, \dots, \mu_p}^{(p)}$ and gravitini $\psi_{\mu\alpha}$. Thus, quantum theory of closed strings are associated with the theory of gravity in spacetime. On the contrary, open strings are topologically equivalent to the interval $[0, \pi]$. When quantized, they give rise to the massless gauge field A_μ in spacetime. On the other hand, supersymmetric open strings have their endpoints on Dirichlet p -branes (Dp-brane) and the associated gauge field lives on the worldvolume of the Dp-brane. Even though the open and closed strings are

physically different, they are associated with each other at the quantum level. Closed strings were first formed through one-loop process of an open string. Accordingly, closed strings were observed as poles in nonplanar one-loop open string diagrams [267, 268]. The degree of freedom of an open string is related to a standing wave modes whilst the closed string is related to left and right-moving waves. There are two boundary conditions for open strings; Neumann (freely moving endpoints) and Dirichlet (fixed endpoints) boundary conditions. The closed strings on the other hand are associated with periodic and an anti-periodic boundary conditions. Some Dp -brane have an unstable configurations both in the supersymmetric or the nonsupersymmetric string theories. The instability is attributed to tachyonic modes with negative square masses $m_\phi^2 < 0$ in an open string spectrum with the end points on the D-brane [269, 270]. Some Dp -brane configurations with open strings containing tachyons are:

- **Brane-Anti-brane**; it is a type IIA or IIB string theory with parallel Dp -branes separated by $d < l_s$ (l_s is the string length). The difference in their orientation leads to opposite Ramond-Ramond (R-R) charges [271], they carry tachyons in their open string spectrum. So, the brane and the anti-brane can annihilate leaving a neutral vacuum because the net R-R charge will be zero.
- **Wrong-dimensional branes**; the Dp -brane with wrong dimension for type IIA/IIB with odd/even spatial dimension for p instead of even/odd dimensions carry no charges under classical IIA/IIB supergravity fields, they also carry tachyons in their open string spectrum. Such branes can annihilate to form vacuum without violating charge conservation.
- **Bosonic Dp -branes**; just as the wrong-dimension branes of type IIA/IIB string theory, the Dp -brane of any particular dimension in the bosonic string theory have no conserved charge and has tachyons in their open string spectrum. Also, they can annihilate to form vacuum without violating charge conservation.

Furthermore, non-BPS Dp -branes of type IIA/IIB string theories are unstable due to the presence of tachyons on its worldvolume [272]. However, a stable non-BPS branes can be developed taking the orientifold/orbifolds of the theory which projects out the tachyons from the system [272, 273, 274].

3.2 Modification of Dirac-Born-Infeld Action

In this section we will introduce the Born-Infeld (BI) action [275], referred to in some literature as the Dirac-Born-Infeld (DBI) action [276, 277]. The theory is grounded on nonperturbative Dirichlet- p -branes [278, 279] on which open strings live. The theory is also coupled with Ramond-Ramond states and other fields which are quanta of close strings. The nonperturbative feature of the branes [280] make them suitable candidates for investigating low energy degrees of freedom and by extension, nonperturbative QCD. The difference between the DBI action and other p -brane and supermembrane theories [271, 281, 282] is the presence of gauge field on the worldvolume of the branes in DBI action. The presence of the gauge field signifies the presence of virtual open string states in the theory, making it possible to study open strings connecting D-branes, open strings connecting antibrane to antibrane and those connecting branes to antibranes. The action for the theory can be expressed as

$$S = -T_p \int d^{p+1} \xi e^{-\varphi} \sqrt{-\det (G_{\mu\nu} + B_{\mu\nu} + (2\pi\alpha') F_{\mu\nu})} + S_{CS} + \text{fermions}; \quad (3.3)$$

where $G_{\mu\nu}$, $B_{\mu\nu}$ and φ are the induced metric tensor, antisymmetric tensor and dilaton field to the Dp -brane worldvolume respectively. In general, there is also fermionic term, $i(2\pi\alpha')^2 \bar{\Theta} \Gamma_\mu \partial_\nu \Theta$, under the square root whose dynamics will be soon incorporated in Sec. 3.4. While $F_{\mu\nu}$ is the worldvolume electromagnetic field strength tensor of the gauge field A_μ , S_{CS} is the set of Chern Simons terms while the brane tension is expressed as

$$\tau_p = \frac{T_p}{g_s} = \frac{1}{g_s \sqrt{\alpha'} (2\pi \sqrt{\alpha'})^p}. \quad (3.4)$$

Also, $g_s = e^{\langle \varphi \rangle}$ is the string coupling, with associated string tension

$$T_{\text{string}} = \frac{1}{2\pi\alpha'}. \quad (3.5)$$

For instance, in IIA/IIB string theory consisting even/odd brane dimensions p , carry A_μ , $\mu = 0, 1, \dots, p$ and X^M , $M = p+1, \dots, 9$ open string quanta, where the scalar fields X^M are the consequence of the gauge field living on the hypersurface of $X^\mu(\xi)$ transverse excitations. Here, the geometry of the Dp -brane is not flat, $X^\mu(\xi)$ defines an embedding, where ξ^μ represent $p+1$ coordinates on the Dp -brane worldvolume $\Sigma_{(p+1)}$, and X^μ is the ten functions mapping from $\Sigma_{(p+1)}$ onto the spacetime manifold $\mathbb{R}^{9,1}$. Bearing in mind that the action must remain invariant under diffeomorphism and gauge transformations,

we fix the degrees of freedom using a 'static gauge' thus,

$$X^\mu \equiv \xi^\mu \quad \text{for} \quad 0 \leq \mu \leq p, \quad (3.6)$$

and the remaining fields are

$$X^\mu \equiv X^M \quad \text{for} \quad (p+1) \leq M \leq d-p; \quad (3.7)$$

where X^M is the transverse coordinate on the worldvolume [283, 284, 285] and d is the spatial dimension. The spatial dimension depends on the theory under study, for instance, $d = 9$ and $d = 25$ correspond to superstring and bosonic string theories respectively. With this static gauge, the initial $d+1$ -dimensional global poincaré symmetry spontaneously breakdown into a product of $p+1$ -dimensional poincaré and $d-p$ -dimensional rotation groups, i.e. $\text{SO}(1, d) \rightarrow \text{SO}(1, p) \times \text{SO}(d-p)$. Consequently, a Dp -brane with $p < d$, will have p -dimensional hyperplanes inside a d -dimensional space. Moreover, the static gauge leads to two possible consistent truncations;

- $X^M = 0$; gives rise to BI theory [275] in $\mathbb{E}^{p,1}$ and
- $F_{\mu\nu} = 0$; leads to Dirac's theory [277] of minimal timelike sub-manifolds of $\mathbb{E}^{d,1}$.

Now, we can introduce the scalar fields into the action through the induced metric tensor

$$G_{\mu\nu} \approx \eta_{\mu\nu} + \eta_{MN} \partial_\mu X^M \partial_\nu X^N, \quad (3.8)$$

this approximates the geometry of the branes to near flat with 10-dimensional Minkowski spacetime metric $\eta_{\mu\nu}$.

The objective of the current study is to investigate the dynamics of the bosonic open strings with tachyonic modes in their spectrum. Accordingly, we will consider a non-BPS brane in IIA/IIB string theory with a massless $U(1)$ vector field, transverse oscillating fields X^M and a tachyonic field T on its worldvolume [287]. Considering that the massless particles are slowly varying relative to the string length l_s i.e. higher order derivatives of the fields can be ignored in the analysis. Hence, the DBI action defined in Eq.(3.3) [280, 286, 288] can be simplified by decoupling S_{CS} and fermionic fields

$$S = -T_p \int d^{p+1} \xi e^{-\varphi} \sqrt{-\det (\eta_{\mu\nu} + \eta_{MN} \partial_\mu X^M \partial_\nu X^N + B_{\mu\nu} + (2\pi\alpha') F_{\mu\nu})}. \quad (3.9)$$

We can now modify the above action by introducing the dynamics of the tachyons into the system similar to the study in [286]. Consequently,

$$S = -T_p \int d^{p+1} \xi e^{-\varphi} V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \eta_{MN} \partial_\mu X^M \partial_\nu X^N + B_{\mu\nu} + (2\pi\alpha') F_{\mu\nu})}; \quad (3.10)$$

here, $V(T)$ is the potential of the tachyon and $\partial_\mu T \partial_\nu T$ is its kinetic energy [289]. The action vanishes at the minimum of the tachyon potential $V(T_0) = 0$ [31, 290, 291]. Since we are interested in investigating open strings at tachyonic vacuum, we remove all the close string quanta, hence, we assume that the geometry of the brane is flat, constant dilaton and vanishing antisymmetric two-form term $B_{\mu\nu}$ in the subsequent analyses.

3.2.1 Dimensional Reduction U(1) Yang-Mills Theory

Investigating Dp -branes using Yang-Mills theory brings to light the physics of Dp -branes without necessarily employing any complicated string theory artefacts. Recent studies indicates that there are enough basics to say that super Yang-Mills theory contains much information on string theory than one may possibly imagine [292]. Also, current developments in high energy physics investigations established that string theory gives an impressive insight into low energy nonperturbative field theory [284]. It has since been conjectured that this behaviour is in agreement with the regime of QCD theory where *color confinement* is observed. In the limit where all the close string quanta and the antisymmetric terms decouple from the theory, we can expand Eq.(3.9) as

$$S = -\tau_p V_p - \frac{1}{4g_{YM}^2} \int d^{p+1} \xi \left(F_{\mu\nu} F^{\mu\nu} + \frac{2}{(2\pi\alpha')^2} \partial_\mu X^M \partial_\nu X_M \right) + \mathcal{O}(F^4). \quad (3.11)$$

Here, V_p is the p -brane worldvolume and g_{YM} is the Yang-Mills coupling derived from

$$g_{YM}^2 = \frac{1}{(2\pi\alpha')^2 \tau_p} = \frac{g_s}{\sqrt{\alpha'}} (2\pi\sqrt{\alpha'})^{p-2}. \quad (3.12)$$

The second term in Eq.(3.11) corresponds to U(1) gauge theory in $p+1$ dimension coupled with $9-p$ scalar fields. Hence, the scalar fields gets decoupled automatically when we choose $p = 9$, hence, introducing fermions into the action leads to the recovery of $\mathcal{N} = 1$ supersymmetric U(1) Yang-Mills theory in $10d$,

$$S = \frac{1}{g_{YM}^2} \int d^{10} \xi \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\psi} \Gamma^\mu \partial_\mu \psi \right). \quad (3.13)$$

When we have N Dp -branes with p -dimensions arranged parallel to each other, they must be distinctly labelled from 1 to N . Fast forward, massless vector fields living on the

worldvolume of each brane are collectively represented by $U(1)^N$ gauge group. Thus, the fields connecting one string to the other on the worldvolume is labelled as $A_{i,j}^\mu$, where i, j distinguishes the branes carrying the endpoints of the strings. The open strings are oriented such that they carry a massless set of $N(N-1)$ fields generally, $A_\alpha = (A_\mu, X^M)$, made up of the Abelian worldvolume gauge tangential to the brane and the transverse scalar field. The mass of each string is proportional to its length therefore, the strings become massless [279, 293] when the branes are closely packed together. Consequently, the opened strings transform under the adjoint representation of $U(N)$, which can further be decomposed as $U(N) = SU(N) \times U(1)$ in $p+1$ -dimensions. Considering stacks of Dp -branes intersecting each other at an angle, we can decompose $U(N)$ into Standard Model (SM) symmetry $SU(3) \times SU(2) \times U(1)$ [294]. In conclusion, DBI action can be generalized to investigate particles that fall under non-Abelian gauge group, considering stacks of Dp -branes.

The duality in string and gauge theories are motivated by the 't Hooft large N_c -limit [295, 296]. Even though, $SU(N_c)$ and $U(N_c)$ groups have distinct representations, in the limit $N_c \rightarrow \infty$, the differences can be ignored. In this range, the number of gluons present can be approximated as $\sim N_c^2$. This surpasses the number of degrees of freedom of quarks, $N_f N_c$, hence, the dynamics of the gluons are dominant here [297].

3.3 Bosonic Opened Strings at Tachyonic Vacuum

Generally, the Dirichlet and Neumann boundary conditions are well grounded in literature [280, 298, 299]. With regards to Dirichlet boundary condition, a small displacement normal to the string and the brane will reflect back with a phase shift of π [300]. This has been applied to Nambu-Goto action with a background 3-branes [301, 302]. Since the endpoints of the strings connecting the 3-branes behave like electric charges, Neumann boundary condition, where the endpoints of the strings are freely oscillating will bring about the creation of an electromagnetic dipole radiation at the asymptotic outer area of the brane.

In this section, we proceed with Eq.(3.10) in the limit of vanishing $B_{\mu\nu}$, with tachyons in the open string spectrum. However, we will reduce the 10-dimensional space-time contained in superstring theory to 4-dimensional spacetime. This is necessary be-

cause, we intend to use the theory to describe SM particles known to be living on 4-dimensional spacetime. The major difference between superstring and elementary particle physics is that, the former lives on 10-dimensions and the latter lives on 4-dimensions. The discrepancy can be resolved using compactification scheme [303, 304, 305], where spacetime is divided into external noncompact spacetime $\mathcal{M}_{10-d}$ and an internal compact spacetime $\mathcal{M}_d$. This can be simplified in an expression

$$\mathcal{M}_{10} = \mathcal{M}_{10-d} \times \mathcal{M}_d; \quad (3.14)$$

here, we will proceed by choosing a physically realistic phenomenon where $d = 6$. Furthermore, the compactification process leads to a compact scale $M_c = 1/R$, where R is the radius of the internally compact spacetime whose magnitude is less than the string mass $M_s = 1/l_s$ [41, 306]. Accordingly, the energy E required for the study lies within $E \ll M_c \ll M_s$, which is suitable for the analysis. Also, the worldvolume coordinate becomes $\xi = (x^\mu, x^6)$, $\mu = 0, 1, 2, 3$, thus, we have 1+3-dimensional external noncompact spacetime with $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$ metric signature. Again, the $d - p$ scalar fields get decoupled leading to the expression

$$\begin{aligned} S &= -\tau_p \int d^4x V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + (2\pi\alpha') F_{\mu\nu})} \\ &= -\tau_p \int d^4x V(T) \sqrt{1 - \eta^{\mu\nu} \partial_\mu T \partial_\nu T + \frac{1}{2}(2\pi\alpha')^2 F_{\mu\nu} F^{\mu\nu} + \dots} \\ &= -\tau_p \int d^4x V(T) \left[1 - \frac{1}{2} \partial_\mu T \partial^\mu T + \frac{1}{4} (2\pi\alpha')^2 F_{\mu\nu} F^{\mu\nu} + \dots \right]. \end{aligned} \quad (3.15)$$

The second step is a good approximation if we consider that the second and higher order derivative terms of T and A_μ are small. In the last step, we have assumed that the tachyons are slowly varying, so $(2\pi\alpha')^2 F_{\mu\nu} F^{\mu\nu}$ and $\partial_\mu T \partial^\mu T$ are relatively small and of the same order. This represents a modification of Eq.(3.3), where the open strings can be studied at tachyonic vacuum. It has been conjectured that open bosonic strings studied at tachyonic vacuum behaves as if they are close strings with no Dp -branes. However, soliton solutions in this region points to the presence of lower dimensional branes [31, 32]. This projection is also corroborated in superstring theories [290, 272] and evident in first [291, 274, 307] and second [49, 308, 309, 310, 311, 312, 313, 314] quantizations in string theories [315]. At this point, the strings behave like close strings, because their endpoints are connected by a flux-line on the brane. Also, the endpoints of the strings on the remaining pieces of the brane behave like point-like particles representing the source and

sink of the flux carrying the color particles. This is expected because, in this region, the negative energy density of the tachyons $V(T_0)$ cancel [293] the tension of the branes τ_p , i.e. $V(T_0) + \tau_p = 0$, so there is no energy lost in rearranging the strings on the worldvolume of the branes [264, 316, 115].

Now we assume that the tachyons can be rewritten in terms of another particle field say, ϕ , which yields to the transformed tachyon potential

$$V(T(\phi)) = \left(\frac{\partial \phi}{\partial T} \right)^2, \quad (3.16)$$

therefore,

$$\frac{1}{2}V(T)\partial_\mu T\partial^\mu T = \frac{1}{2}\left(\frac{\partial \phi}{\partial T}\frac{\partial T}{\partial x}\right)^2 = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi. \quad (3.17)$$

Consequently, the Lagrangian density derived from Eq.(3.15) with respect to ϕ , reads,

$$\tau_p^{-1}\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4}G(\phi)F_{\mu\nu}F^{\mu\nu}. \quad (3.18)$$

We have defined a dimensionless quantity $G(\phi) = (2\pi\alpha')^2 V(\phi)$ which will be called 'color dielectric function' in the rest of the analysis. For simplicity, we will set $(2\pi\alpha') = 1$, leading to a string tension $T_{string} = 1$, and $G(\phi) = V(\phi)$ [1, 2, 317]. It is important to mention that the potential of the field described by ϕ follows the same discussions as in Sec. 3.1.1. It has its minimum at $V(\phi = \langle\phi\rangle_0) = 0$, where $\langle\phi\rangle_0$ is the true minimum of the potential and an extremum at $\langle\phi\rangle_0 = 0$. To describe an asymptotically free system, the potential must satisfy additional conditions

$$V(\phi = \langle\phi\rangle_0) = 0 \quad , \quad \frac{\partial V}{\partial \phi}|_{\phi=\langle\phi\rangle_0} = 0 \quad \text{and} \quad \frac{\partial V}{\partial \phi}|_{\phi=0} = 0. \quad (3.19)$$

These conditions are necessary to stabilize the vacuum of an asymptotically free systems [1, 2, 318, 319]. When all these conditions are followed to select a suitable potential, the theory Eq.(3.18) will be viable for investigating color particles such as quarks and gluons in QCD-like fashion [317, 66]. In that light, we treat the endpoints of the open string connecting the branes as a source of valence gluons and the string connecting them as gluons that mediate the interactions. In the presence of fermions, the source are quark and an anti-quark mediated by gluons. Additionally, the potential density is analogous to the quantum correction required in gluodynamics theory to break the conformal symmetry and to bring about nonvanishing gluon condensation, a phenomenon which is important in QCD theory [318, 320].

To extend the theory to include a system of Dp -brane and an anti- Dp -brane, we ensure that the theory is invariant under global rotation symmetry, $\phi \rightarrow e^{i\alpha}\phi$. Hence, we introduce a complex scalar field ϕ [321],

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}. \quad (3.20)$$

To fulfill these conditions, we need to impose a gauge invariant condition on the scalar sector by introducing an *Abelian covariant derivative*,

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - iq\tilde{A}_\mu; \quad (3.21)$$

where $\tilde{A}_\mu$ is a gauge field dual to A_μ . Consequently, the Lagrangian density becomes,

$$\tau_p^{-1}\mathcal{L} = D_\mu\phi D^\mu\phi^* - V(|\phi|) - \frac{1}{4}G(|\phi|)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}; \quad (3.22)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ are two independent Abelian field strengths. Beside the $U(1)$ gauge invariance of the Lagrangian, it is also invariant under the $\tilde{U}(1)$ gauge transformations,

$$\phi(x) \rightarrow \phi'(x) = e^{-iq\alpha(x)}\phi \quad \text{and} \quad \tilde{A}_\mu(x) \rightarrow \tilde{A}'_\mu(x) = \tilde{A}_\mu(x) - \partial_\mu\alpha(x). \quad (3.23)$$

This Lagrangian can undergo a spontaneous symmetry breaking (SSB) process similar to the usual Abelian Higgs mechanism [123, 124] due to the high symmetry in ϕ . This process will also lead to the observation of Goldston boson (most likely π^0 due to the $\tilde{U}(1)$ global symmetry) corresponding to the number of unbroken symmetries, signalling *color confinement* [3, 322].

3.4 Modified Gauge Theories with Fermions

In this section, we will analyse how the theories derived above can be systematically coupled with SM fermions following gauge invariant principles. We will also explore how non-Abelian version of the theory can be constructed with fermion couplings. This will enable us discuss color confinement of quarks and gluons in a comprehensive manner.

3.4.1 Fermions Coupled to Fundamental Strings at Tachyonic Vacuum

In this section we introduce the Dirac's Lagrangian for free particles adopted by Maxwell in unification of electric and magnetic interactions. An important feature of

Maxwell's equation is that, it has no specific electromagnetic potential that generate fields. Consequently, there is freedom in selecting different kinds of potentials to study the same electromagnetic fields, this is generally referred to as *gauge invariance*. Let us start with Dirac's free particle Lagrangian

$$\mathcal{L}_0 = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi; \quad (3.24)$$

where ψ and $\bar{\psi}$ are the fermion fields, with mass m . Even though, this Lagrangian is well known in QED, it is also used to describe free nucleons in terms of their composite fermions, protons and neutrons in strong interactions. It is invariant under global phase rotation

$$\psi \rightarrow \psi' = e^{i\alpha} \psi. \quad (3.25)$$

Bearing in mind that the fields in the Lagrangian are color neutral, we modify the fields with the *color dielectric function* introduced in Sec. 3.1, to make them suitable for analysing color particles. Thus,

$$\psi \rightarrow \psi' \equiv G^{1/2} \psi \quad \text{and} \quad \bar{\psi} \rightarrow \bar{\psi}' \equiv \bar{\psi} G^{1/2}; \quad (3.26)$$

where G is a function, so the gradient transforms as

$$\partial_\mu \psi \rightarrow \partial_\mu \psi' = [(\partial_\mu G^{1/2}) + G^{1/2} \partial_\mu] \psi, \quad (3.27)$$

hence, the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_0 &= \bar{\psi}' [i\gamma^\mu \partial_\mu - m] \psi' \\ &= \bar{\psi} [i\gamma^\mu G \partial_\mu + i\gamma^\mu G^{1/2} (\partial_\mu G^{1/2}) - mG] \psi; \end{aligned} \quad (3.28)$$

here, *local gauge invariance* is violated by $G(\phi)$ and the gradient function $(\partial_\mu G^{1/2})$. So to ensure that local gauge invariance is satisfied, we modify all observables involving derivatives in Eq.(3.24) with the electromagnetic field $A_\mu(x)$, and also modify the Dirac γ -matrix with $G(\phi)$. Consequently,

$$\gamma^\mu \rightarrow \gamma'^\mu = G^{-1} \gamma^\mu \quad \text{and} \quad \partial_\mu \rightarrow D_\mu \equiv \partial_\mu - iqA_\mu; \quad (3.29)$$

where D_μ is the *gauge-covariant derivative* and q is the electric charge. This kind of derivative leads to the modification of the momentum, $p_\mu \rightarrow p_\mu - qA_\mu$. This will bring about a

coupling between the electromagnetic field and matter in a form $D_\mu\psi$. Accordingly,

$$\begin{aligned}\mathcal{L}_0 &= \bar{\psi} \left[i\gamma^\mu \partial_\mu + i\gamma^\mu \partial_\mu (\ln G^{1/2}) + qA_\mu \gamma^\mu - mG \right] \psi \\ &= \bar{\psi} [i\gamma^\mu \partial_\mu + q\gamma^\mu A_\mu - mG] \psi,\end{aligned}\tag{3.30}$$

as a result, the electromagnetic field transforms as

$$A_\mu \rightarrow A'_\mu \equiv A_\mu + \frac{i}{q} \partial_\mu (\ln G^{1/2}).\tag{3.31}$$

Equation (3.30) is the Dirac Lagrangian with interaction term that couples the gauge field $A_\mu(x)$ to the conserve current $j^\mu = q\bar{\psi}\gamma^\mu\psi$ with a modified mass term $M(r) = mG(\phi)$. To obtain a full Lagrangian suitable for the analysis, we need to derive the kinetic term of the gauge field. However, this has been derived in Eq.(3.18) with intermediate tachyonic field ϕ , therefore, combining Eqs.(3.18) and (3.30) yields,

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu + q\gamma^\mu A_\mu - mG(\phi)) \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} G(\phi) F_{\mu\nu} F^{\mu\nu}.\tag{3.32}$$

This expression looks like the usual QED Lagrangian (omitting gauge-fixing terms for simplicity) with an intermediate tachyonic field ϕ and a modified mass term. Hence, this results serves as an approximation of the non-Abelian gauge theory responsible for strong interaction with an Abelian one [1, 2, 3]. It also follows the long standing projection that the confining regime of strong interaction is mostly Abelian dominant [60, 61, 62, 63, 65]. In this expression, the quark and an anti-quark in the system are represented by the spinor fields while the gluons are represented by A_μ . All the long distance behaviour of the gluons are absorbed by the color dielectric function, so in the presence of $G(\phi)$ the photon propagator obtained from the Abelian gauge field do not decouple at longer wave lengths making it possible to describe gluons. The modified mass function $M(r)$ can be measured at the beginning $r \rightarrow r_0$ and at the end $r \rightarrow r_*$ of the interactions. Therefore, the dielectric function should have a finite value at the beginning and the end of the interactions i.e. $G(\phi(r \rightarrow r_0)) = G(\phi(r \rightarrow r_*)) = \text{constant}$, where $1/r_0$ is the energy at the beginning of the interaction and $1/r_*$ is the energy at the end of the interaction. Hence, $M(r) = mG(\phi)$ represents the *constituent quark mass* of the system [2, 3], whose magnitude depends on the 'bare' quark mass m .

Moreover, the *renormalization theory* remains the systematic approach for which UV divergences can be dealt with [323] in QFT. Accordingly, we juxtapose the results

obtained in Eq.(3.32) with the renormalized QED Lagrangian to identify the characteristics of the dielectric function in the context of renormalization factor $Z(\mu)$, where μ is a scale that comes from the dimensional regularization scheme [324, 325]. This will enable us ensure that the theory is free from UV divergences. Expressing the QED Lagrangian with renormalized factors

$$\mathcal{L}_{\text{QED}} = Z_\psi \bar{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi - \frac{Z_A}{4} F^{\mu\nu} F_{\mu\nu} - Z_1 e \bar{\psi} \gamma_\mu A^\mu \psi; \quad (3.33)$$

where

$$\psi_0 = Z_\psi^{1/2} \psi, \quad A_0^\mu = Z_A^{1/2} A^\mu, \quad m_0 = \frac{Z'_\psi}{Z_\psi} m \quad \text{and} \quad e = \frac{Z_\psi}{Z_1} Z_A^{1/2} e_0. \quad (3.34)$$

Additionally, noting that the unrenormalized mass m_0 consists of both finite mass m and an infinite mass δm i.e. $m_0 = m - \delta m$, we can express the mass renormalization constant Z'_ψ as

$$\begin{aligned} Z'_\psi &= Z_\psi \frac{m_0}{m} \\ &= Z_\psi \frac{m - \delta m}{m} = \left(1 - \frac{\delta m}{m}\right) Z_\psi, \end{aligned} \quad (3.35)$$

then we can compare the above expression (3.33) with Eq.(3.32). The renormalized covariant derivative can also be expressed in terms of renormalization factors as $D_\mu^{\text{ren}} = \partial_\mu - ie(Z_1/Z_\psi)A_\mu$, so gauge invariance warrants that $Z_1 = Z_\psi$. We replaced the 'conventional' expressions of the renormalization factors Z_2 and Z_3 with Z_ψ and Z_A respectively, and $Z_1 = Z_\psi Z_A^{1/2}$ to make the classification obvious in relation to fermion and gauge fields. The *renormalized fields* are written without subscript '0' [326, 327, 328, 329, 330, 331, 332] and the vice versa. Comparing the Lagrangian, we find that $Z_A = Z_\psi = G$, while the gauge invariance requires that $Z_A = 1$. As a result, despite the color properties of the color dielectric function, it also absorbs the UV divergences of the theory. In this vein, the color dielectric function presents the following characteristics,

$$G(\phi(r)) \rightarrow \begin{cases} 1 & \text{for } r \rightarrow 0; \text{ deconfinement/Coulombic regime} \\ 0 & \text{for } r \rightarrow r_*; \text{ confinement regime} \end{cases}. \quad (3.36)$$

On the other hand, a photon mass term

$$\mathcal{L}_\gamma = \frac{1}{2} M^2(r) A_\mu A^\mu, \quad (3.37)$$

is prohibited, because it will not be invariant under the local gauge invariance transformation Eq.(3.31),

$$A_\mu A^\mu \rightarrow A'_\mu A'^\mu \equiv \left(A_\mu + \frac{i}{q} \partial_\mu (\ln G^{1/2}) \right) \left(A^\mu - \frac{i}{q} \partial^\mu (\ln G^{1/2}) \right) \neq A_\mu A^\mu. \quad (3.38)$$

In conclusion, local gauge invariance accounts for the existence of massless photon [124].

3.4.2 Non-Abelian Gauge Theory

We will now focus on constructing a non-Abelian theory that describes the strong nuclear force, traditionally. Owing to almost equal mass for proton and neutron, charge independence of the nuclear force and other observations, there is an agreement that isospin is conserved in strong interaction among proton and neutron. As in the case of QED, we will base the discussions on the SU(2)-isospin gauge theory introduced by Yang and Mills [333] elaborated by Shaw [334]. Thus, we proceed with Eq.(3.24) as the Lagrangian for free nucleon with composite fermions, protons (p) and neutrons (n),

$$\psi \equiv \begin{pmatrix} p \\ n \end{pmatrix}. \quad (3.39)$$

In this case, the theory is invariant under global spin rotation

$$\psi \rightarrow \psi' = e^{i\vec{\tau} \cdot \vec{\alpha}/2} \psi; \quad (3.40)$$

here, the isospin current $j^\mu = \bar{\psi} \gamma^\mu (\tau/2) \psi$ is conserved. As a result, the proton and the neutron can be treated symmetrically, if we ignore a possible electromagnetic interaction among them. In this respect, the distinction between proton and neutron becomes a matter of arbitrariness and convention.

To make a distinction between the Abelian theory discussed in Sec. 3.4.1 and its non-Abelian counterpart intended for this subsection, we will substitute A_μ contained in *Abelian covariant derivative* D_μ with its non-Abelian counterpart B_μ , therefore,

$$D_\mu \equiv \partial_\mu - igB_\mu. \quad (3.41)$$

We have also replaced q with g , the strong coupling constant, to make the analysis distinct. Despite the similar global gauge invariance satisfied by both theories, they also exhibit some major differences; the algebra of the non-Abelian group is more complex and the associated gauge bosons interact among themselves due to the structure of the

non-Abelian gauge group. It is important to mention at this point that, the operators that will be introduced here are 2×2 matrices in the isospin space. Consequently,

$$B_\mu = \frac{1}{2} \vec{\tau} \cdot \vec{b}_\mu = \frac{1}{2} \tau^a b_\mu^a = \frac{1}{2} \begin{pmatrix} b_\mu^3 & b_\mu^1 - ib_\mu^2 \\ b_\mu^1 + ib_\mu^2 & -b_\mu^3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} b_\mu^3 & \sqrt{2} b_\mu^+ \\ \sqrt{2} b_\mu^- & -b_\mu^3 \end{pmatrix}, \quad (3.42)$$

the three gauge fields are $\vec{b}_\mu = (b_\mu^1, b_\mu^2, b_\mu^3)$, the associated generators are τ^a ($a = 1, 2$ and 3), which are related to the Pauli's matrices, and $b_\mu^\pm = (b_\mu^1 \mp ib_\mu^2)/\sqrt{2}$ are the charged bosons. The isospin step-up and step-down operators $1/2(\tau_1 \pm i\tau_2)$ exchange $p \leftrightarrow n$ following the absorption of $b_\mu^\pm$ boson. Thus, the gradient in Eq.(3.24) transforms according to Eq.(3.26) as

$$\begin{aligned} D_\mu \psi &\rightarrow D_\mu \psi' = G^{1/2} \partial_\mu \psi + (\partial_\mu G^{1/2}) \psi - ig B_\mu (G^{1/2} \psi) \\ &= G^{1/2} (\partial_\mu - ig B'_\mu) \psi, \end{aligned} \quad (3.43)$$

in this regard,

$$\begin{aligned} -ig G^{1/2} B'_\mu &= -ig B_\mu G^{1/2} + (\partial_\mu G^{1/2}) \rightarrow \\ B'_\mu &= G^{1/2} B_\mu G^{-1/2} + \frac{i}{g} (\partial_\mu G^{1/2}) G^{-1/2} \\ &= G^{1/2} \left(B_\mu + \frac{i}{g} G^{-1/2} (\partial_\mu G^{1/2}) \right) G^{-1/2}. \end{aligned} \quad (3.44)$$

Nonetheless, adopting similar procedure as in Eq.(3.30) yields,

$$\mathcal{L}' = \bar{\psi} [i\gamma^\mu \partial_\mu + g B_\mu \gamma^\mu - mG] \psi, \quad (3.45)$$

with similar transformation as Eq.(3.31),

$$B_\mu \rightarrow B'_\mu \equiv B_\mu + \frac{i}{g} (\partial_\mu \ln G^{1/2}), \quad (3.46)$$

juxtaposing Eq.(3.44) and Eq.(3.46) leads to

$$B_\mu \rightarrow B'_\mu = G^{1/2} B_\mu G^{-1/2}. \quad (3.47)$$

The generators for SU(2)-isospin group representation follow the commutator relation

$$[\tau^j, \tau^k] = 2i\varepsilon_{jkl} \tau^l. \quad (3.48)$$

Generally, τ_i do not commute with others in different spatial directions, so only one component can be measured at a time and this is usually taken to be the third component $T_3 = 1/2\tau_3$, conventionally in the z -direction.

We will now develop the field-strength tensor that will constitute the kinetic term of the gauge field, starting with the electromagnetism gauge

$$F_{\mu\nu} = \frac{1}{2} \mathbf{F}_{\mu\nu} \cdot \boldsymbol{\tau} = \frac{1}{2} F_{\mu\nu}^a \tau^a \quad \text{where} \quad \text{tr}(\tau^a \tau^b) = 2\delta^{ab}. \quad (3.49)$$

From this transformation we can construct the gauge invariant term

$$\mathcal{L}'_{\text{gauge}} = -\frac{1}{4} \mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu} = -\frac{1}{2} \text{tr}(F_{\mu\nu} F^{\mu\nu}), \quad (3.50)$$

the field-strength tensor is such that it transforms under the dielectric function

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = G^{1/2} F_{\mu\nu} G^{-1/2}. \quad (3.51)$$

We observe from Sec. 3.4.1 that the gauge field-strength can be expressed as

$$\begin{aligned} F'_{\mu\nu} &= \partial_\nu B'_\mu - \partial_\mu B'_\nu = \partial_\nu \left[G^{1/2} B_\mu G^{-1/2} + \frac{i}{g} (\partial_\mu G^{1/2}) G^{-1/2} \right] - \partial_\mu \left[G^{1/2} B_\nu G^{-1/2} + \frac{i}{g} (\partial_\nu G^{1/2}) G^{-1/2} \right] \\ &= \left\{ (\partial_\nu G^{1/2}) B_\mu G^{-1/2} + G^{1/2} (\partial_\nu B_\mu) G^{-1/2} + G^{1/2} B_\mu (\partial_\nu G^{-1/2}) + \frac{i}{g} \left[\partial_\nu (\partial_\mu G^{1/2}) G^{-1/2} + (\partial_\mu G^{1/2}) (\partial_\nu G^{-1/2}) \right] \right\} \\ &\quad - \left\{ (\partial_\mu G^{1/2}) B_\nu G^{-1/2} + G^{1/2} (\partial_\mu B_\nu) G^{-1/2} + G^{1/2} B_\nu (\partial_\mu G^{-1/2}) + \frac{i}{g} \left[\partial_\mu (\partial_\nu G^{1/2}) G^{-1/2} + (\partial_\nu G^{1/2}) (\partial_\mu G^{-1/2}) \right] \right\}, \end{aligned} \quad (3.52)$$

we used the expression for B'_μ expressed in Eq.(3.44). Rearranging the terms in the above expression yields,

$$\begin{aligned} F'_{\mu\nu} &= G^{1/2} [\partial_\nu B_\mu - \partial_\mu B_\nu] G^{-1/2} + [(\partial_\nu G^{1/2}) B_\mu - (\partial_\mu G^{1/2}) B_\nu] G^{-1/2} + \\ &\quad G^{1/2} [B_\mu (\partial_\nu G^{-1/2}) - B_\nu (\partial_\mu G^{-1/2})] + \frac{i}{g} [(\partial_\mu G^{1/2}) (\partial_\nu G^{-1/2}) - (\partial_\nu G^{1/2}) (\partial_\mu G^{-1/2})] \\ &\neq G^{1/2} [\partial_\nu B_\mu - \partial_\mu B_\nu] G^{-1/2}, \end{aligned} \quad (3.53)$$

higher order derivative terms were discarded. We can cast the result in slightly symmetric form by using the identity $G^{1/2} G^{-1/2} = \mathbb{I}$,

$$\begin{aligned} \partial_\mu (G^{1/2} G^{-1/2}) &= (\partial_\mu G^{1/2}) G^{-1/2} + G^{1/2} (\partial_\mu G^{-1/2}) = 0 \rightarrow \\ G^{1/2} (\partial_\mu G^{-1/2}) &= -(\partial_\mu G^{1/2}) G^{-1/2}. \end{aligned} \quad (3.54)$$

Using the above identity leads to

$$\begin{aligned} F'_{\mu\nu} &= G^{1/2} (\partial_\nu B_\mu - \partial_\mu B_\nu) G^{-1/2} + \left((\partial_\nu G^{1/2}) B_\mu - B_\mu (\partial_\nu G^{1/2}) \right) G^{-1/2} + \\ &\quad \left(B_\nu (\partial_\mu G^{1/2}) - (\partial_\mu G^{1/2}) B_\nu \right) G^{-1/2} + \frac{i}{g} \left((\partial_\nu G^{1/2}) G^{-1/2} (\partial_\mu G^{1/2}) - (\partial_\mu G^{1/2}) G^{-1/2} (\partial_\nu G^{1/2}) \right) G^{-1/2} \\ &= G^{1/2} [\partial_\nu B_\mu - \partial_\mu B_\nu] G^{-1/2} + [(\partial_\nu G^{1/2}), B_\mu] G^{-1/2} + [B_\nu, (\partial_\mu G^{1/2})] G^{-1/2} \\ &\quad + \frac{i}{g} [(\partial_\nu G^{1/2}), G^{-1/2} (\partial_\mu G^{1/2})] G^{-1/2} \\ &= G^{1/2} (\partial_\nu B_\mu - \partial_\mu B_\nu) G^{-1/2} + G^{1/2} \left\{ [G^{-1/2} (\partial_\nu G^{1/2}), B_\mu] - [G^{-1/2} (\partial_\mu G^{1/2}), B_\nu] \right\} G^{-1/2} \\ &\quad + \frac{i}{g} G^{1/2} [G^{-1/2} (\partial_\nu G^{1/2}), G^{-1/2} (\partial_\mu G^{1/2})] G^{-1/2}. \end{aligned} \quad (3.55)$$

In the above expression, there are additional terms from the non-vanishing commutators as a result of the non-Abelian group structure. Hence, $F'_{\mu\nu}$ will require some modifications in order to suitably transform under the gauge group. In view of this, the initial Abelian gauge field-strength tensor is modified

$$F_{\mu\nu} = \frac{1}{iq} [D_\nu, D_\mu], \quad (3.56)$$

using D_μ expressed earlier into the above equation,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + iq [A_\nu, A_\mu]; \quad (3.57)$$

where the commutator vanishes for Abelian formalism. Applying this transformation to the case under study Eq.(3.51), we can express

$$F'_{\mu\nu} = \partial_\nu B'_\mu - \partial_\mu B'_\nu + ig [B'_\nu, B'_\mu]. \quad (3.58)$$

Now, we check if the additional terms generated by the commutator actually cancel out the additional non-vanishing commutators in Eq.(3.55) thus, we expand the above commutator

$$\begin{aligned} ig [B'_\nu, B'_\mu] &= ig \left[\left(G^{1/2} B_\nu G^{-1/2} + \frac{i}{g} (\partial_\nu G^{1/2}) G^{-1/2} \right), \left(G^{1/2} B_\mu G^{-1/2} + \frac{i}{g} (\partial_\mu G^{1/2}) G^{-1/2} \right) \right] \\ &= ig G^{1/2} [B_\nu, B_\mu] G^{-1/2} - G^{1/2} \left[B_\nu, G^{-1/2} (\partial_\mu G^{1/2}) \right] G^{-1/2} - \\ &G^{1/2} \left[G^{-1/2} (\partial_\nu G^{1/2}), B_\mu \right] G^{-1/2} - \frac{i}{g} G^{1/2} \left[G^{-1/2} (\partial_\nu G^{1/2}), G^{-1/2} (\partial_\mu G^{1/2}) \right] G^{-1/2} \\ &= ig G^{1/2} [B_\nu, B_\mu] G^{-1/2} - G^{1/2} \left\{ \left[G^{-1/2} (\partial_\nu G^{1/2}), B_\mu \right] - \left[G^{-1/2} (\partial_\mu G^{1/2}), B_\nu \right] \right\} G^{-1/2} \\ &- \frac{i}{g} G^{1/2} \left[G^{-1/2} (\partial_\nu G^{1/2}), G^{-1/2} (\partial_\mu G^{1/2}) \right] G^{-1/2}. \end{aligned} \quad (3.59)$$

The expressions that come after the first term in the last step are the exact terms required to cancel the extra terms in Eq.(3.55). This gives the field-strength tensor the required structure under local gauge transformation. Combining Eqs.(3.45) and (3.50) gives rise to a modified Yang-Mills Lagrangian [124, 333],

$$\mathcal{L}_{\text{YM}} = \bar{\psi} (i\gamma^\mu \partial_\mu + g\gamma^\mu B_\mu - mG) \psi - \frac{1}{2} \text{tr} (F_{\mu\nu} F^{\mu\nu}); \quad (3.60)$$

where $M(r) = mG(\phi)$ is the modified nucleon mass. Additionally, one can introduce the potential and the kinetic energy terms of the intermediate tachyonic field ϕ for the sake of analysis. This Lagrangian is invariant under local gauge transformation and does not permit the existence of a mass term $M^2(r) B_\mu B^\mu$. Using the expressions in Eq.(3.42) and

the commutator relation in Eq.(3.48), we can express

$$\begin{aligned}
F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + ig [B_\mu, B_\nu] \rightarrow \\
F_{\mu\nu}^l &= \frac{1}{2} \left(\partial_\mu (\tau^l b_\nu^l) - \partial_\nu (\tau^l b_\mu^l) \right) + \frac{ig}{4} \left(-2i\varepsilon_{jkl} (\tau^l b_\nu^j b_\mu^k) \right) \\
&= \partial_\mu b_\nu^l - \partial_\nu b_\mu^l + g\varepsilon_{jkl} b_\nu^j b_\mu^k;
\end{aligned} \tag{3.61}$$

in the last step we have dropped the three isospin generators τ^a of the gauge field because they are linearly independent. Generally, gauge groups that do not fall under SU(2), the Levi-Civita symbol ε_{jkl} is replaced with the *antisymmetric structure constant* f_{jkl} .

Even though, SU(2)-isospin symmetry group was pioneered by Heisenberg [335] at a time the quark model was not yet conceived however, the only known fundamental particles found in the nucleon are the up (u) and down (d) quarks, making it possible to be represented under this group. The proton and the neutron that constitute the nucleon are made up of these fundamental particles in different combinations. Under this symmetry group, the proton and the neutron or the u and d quarks are treated as the same particles, with equal mass, same coupling force, but different representations, neglecting possible electromagnetic interactions. Additionally, the proton is composed of two u -quarks and a d -quark and the neutron is also composed of two d -quarks and an u -quark. Accordingly, Eq.(3.60) can be used to study the characteristics of quarks and gluons inside the hadron (proton and neutron). With regard to the fundamental particles, the up/down quarks are treated as having 0-charge with different isospin states, $I_{3u/d} = \pm 1/2$, equivalent to the nucleon. Besides, the spin addition of the quark constituents that form proton and neutron agree with $I_3 = 1/2$ and $I_3 = -1/2$, respectively. Similar to the isospin representation of the nucleon field ψ in Eq.(3.39), the wave function for the quark fields are represented by the SU(2) flavor group

$$\psi_q \equiv \begin{pmatrix} u \\ d \end{pmatrix}. \tag{3.62}$$

Nevertheless, the mathematical structure of the theory is the same as that of quantum chromodynamics (QCD), the theory that describes the characteristics of quarks and gluons inside the hadrons. Acknowledging that the strong interaction is invariant under quark

color permutations, we can express the quark wave function for color triplet representation

$$\psi_c \equiv \begin{pmatrix} R \\ G \\ B \end{pmatrix}; \quad (3.63)$$

where $R \equiv \text{red}$, $G \equiv \text{green}$ and $B \equiv \text{blue}$. This is invariant under $SU(3)$ color group representation

$$\psi \rightarrow \psi' \equiv \exp\left(i\frac{1}{2}\lambda_j\alpha_j\right)\psi; \quad (3.64)$$

where α_j , $j = 1, 2, 3, \dots, 8$ are the eight phase angles and λ_j which is a 3×3 matrices are the eight independent traceless Hermitian generators of the group. Supposing that the differences in the masses of u , d , and s quarks can be ignored, the wave function can be expressed by the $SU(3)$ flavor group

$$\psi_q \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}. \quad (3.65)$$

The generators that appear here are fundamentally, 3×3 matrix equivalent to the Pauli's matrices for the $SU(2)$ representation, they satisfy the Lie algebra

$$[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k, \quad (3.66)$$

similar to Eq.(3.48), f_{ijk} is the *structure constant* [123].

In conclusion, the theory is expected to demonstrate the two most important characteristics of strong interactions i.e., *asymptotic freedom* [16, 17] and *color confining* behaviour. At the low energies where the *color confinement* is expected to set in, here classical description of quantum fields are not tenable. In the framework of the theory, $M(r)$ represents the *constituent quark mass* function [336] and m is the *bare quark mass*. As discussed in Sec. 3.4.1, the constituent quark mass in the asymptotically free region (high energy) can be determined, $M(r \rightarrow r_0) = M_0$, while the mass at the nonperturbative region (low energy) where *color confinement* is expected will be $M(r \rightarrow r_*) = M_*$. Hence, it is possible to have the same *constituent quark mass* in both the UV ($r \rightarrow r_0$) and the IR ($r \rightarrow r_*$) regions if the *bare quark mass* m in both regions are the same [56, 1]. Because the color dielectric function $G(\phi)$, behaves as $G(\phi(r \rightarrow r_0)) = G(\phi(r \rightarrow r_*)) = \text{constant}$. However, it has been established that in order to obtain confinement in both IR and UV

regions, m will have relatively small magnitude in the IR region and higher in the UV region [2, 3]. Accordingly, to observe *color confinement* in both regions, $M_0 > M_*$, because higher *bare quark mass* m is required to obtain confinement in the UV region than it is required in the IR region.

3.5 Analysis

In this chapter we developed the theories that will be used to build models in subsequent chapters. The DBI action essentially treats an opened string on a Dp -brane. The objective is to study the characteristics of the string at a tachyonic vacuum, where the open strings behave like close ones. It has been conjectured that in this region, the negative energy density of the tachyons exactly cancel the brane tension. Consequently, the string is connected by a flux-line on the brane forming a close string, because the brane dissolves into lower dimensional ones living at the endpoints of the string. To apply the theories in QCD phenomenology, the endpoints of the string on the remaining brane services as the source and sink of the color charge. Additionally, to make a case for applying these theories to strong interactions, we derived $10d$ $U(1)$ Super Yang-Mills theory from the DBI action through small perturbation. That notwithstanding, it can be generalized to study a non-Abelian behaviour of the color particles considering stacks of Dp -branes. The theory is initially built on 10-dimensional spacetime however, to make it suitable for describing SM fields, we compactified the internal 6-dimensions leaving 4-dimensions for the analysis.

Furthermore, we demonstrated how SM fermions can be introduced into the theories through systematic approach. This will enable us investigate strong interactions among fermions, since the original theory is based on bosons. Moreover, since pions can exist as either relativistic or nonrelativistic particles with an integer spin which are not governed by Pauli's exclusion principle, they can be described by spin-0 fields similar to the tachyonic field ϕ . In the relativistic regime, they exist as an unstable particles whilst in the nonrelativistic regime they appear as Goldstone bosons [337]. Physically, pions are produced through matter annihilation of $p\bar{p}$, $N\bar{N}$, ee^+ ect. which undergo structural transformation from baryon to mesons [338, 339]. Consequently, the tachyon field ϕ can be used to represent pions, π^+ , π^0 and π^- . In that case, the scalar field can be expressed

as

$$\phi \equiv \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \quad (3.67)$$

which transform under $SU(2) \sim SO(3)$ group

$$\phi \rightarrow \phi' = e^{i\alpha^a t_a} \phi. \quad (3.68)$$

On the other hand, the complex scalar field

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}, \quad (3.69)$$

can be used to study a system of charged pions, $\pi^\pm$. Also, the complex scalar field defined above can be represented under the rotation symmetry group since $U(1) \sim SO(2)$,

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \phi' = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad (3.70)$$

to study two identical particles, most likely charged pions. So the particles/fields under study can undergo spontaneous symmetry breaking process which leads to the release of Goldstone bosons similar to Higgs mechanism due to the Z_2 symmetry. Better still, the potential governing these particles/fields involved can be shifted and expanded around the true minimum to removed the tachyonic modes in the spectrum and to stabilize the vacuum. This process signals the breakdown of chiral symmetry due to the small mass possessed by the u and d quarks that constitute the Goldstone bosons. This will naturally lead to color confinement. Since pions are the secondary most abundant particles produced in heavy ion collisions, proper understanding will aid the search for *physics beyond the standard model*. Besides, they are particles which can interact among themselves through strong, electromagnetic and weak forces.

Chapter 4

Confinement of Glueballs

In this chapter we will study the phenomenon of *color confinement* using the tachyon Lagrangian developed in Eq.(3.18). This phenomenon will be based on confining the electric field, i.e. *chromoelectric flux* confinement. The *Flux tube* models are built on the assumption that the confined quark and an anti-quark pairs are static, so the *chromomagnetic field* is completely eliminated in order not to induce spin-orbit interactions that will generate *chromomagnetic flux*. This phenomenon was highlighted in Sec. 2.7 under Maxwell's equations. The electric field and the *color dielectric function* derived in Eqs.(2.86) and (2.93) will be important for this investigations. Besides, we will use the Abelian theory to approximate the non-Abelian theory suitable for investigating strong interactions. We are inspired by an Abelian dominance and observation that about 92% of the QCD string tension is Abelian [54, 64, 62]. Also, 't Hooft projected that non-Abelian QCD theory can be approximated to an Abelian theory with the formation of magnetic monopoles that further condenses to form *color confinement* and *color superconductivity* [60, 61]. Again, results for QCD lattice calculations on SU(2) and SU(3) gauge groups confirm the *Abelain dominance* [63]. Additional detail reviews on *Abelain dominance* can be found in [65].

Furthermore, the principle of this model is similar to 'The Bag Model' discussed in Sec. 2.3.1 where the dielectric function [53, 54] $G(\phi)$ behaves like a QCD vacuum (2.87) and the hadron as a perfect conductor. The dielectric function also absorbs the long distance dynamics of the theory to bring about confinement. Thus, the coupling of the dielectric function to the gauge field $F^{\mu\nu}F_{\mu\nu}$ ensures that only low energy components are present for interactions. Accordingly, the dielectric function ensures that the photon

propagator do not decouple at long distance separation where the gluons are expected to be in a confined state [2].

4.1 The Model

Starting with the Lagrangian density (3.18)

$$\tau_p^{-1}\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}G(\phi)F_{\mu\nu}F^{\mu\nu} - V(\phi). \quad (4.1)$$

The equations of motion are

$$\partial_\mu\partial^\mu\phi + \frac{1}{4}\frac{\partial G(\phi)}{\partial\phi}F_{\mu\nu}F^{\mu\nu} + \frac{\partial V}{\partial\phi} = 0, \quad (4.2)$$

and

$$\partial_\mu[G(\phi)F^{\mu\nu}] = 0. \quad (4.3)$$

We are interested in computing static confining potentials, so we define the indices such that only the static sector of the scalar field is available for the analysis thus, $\mu = 1, 2, 3$ and $j^\nu = (\rho, \vec{0})$. In spherical coordinates Eq.(4.2) reads

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = -\frac{1}{2}\frac{\partial G(\phi)}{\partial\phi}E^2 + \frac{\partial V(\phi)}{\partial\phi}; \quad (4.4)$$

where $F^{0j}F_{0j} = -2E^2$ and $V = G$. Hence, we obtain

$$\begin{aligned} \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) &= \frac{\partial}{\partial\phi}\left[\frac{\lambda^2}{2}\frac{1}{V(\phi)}\frac{1}{r^4} + V\right] \rightarrow \\ \nabla^2\phi &= \frac{\partial}{\partial\phi}\left[\frac{\lambda^2}{2}\frac{1}{V(\phi)}\frac{1}{r^4} + V\right], \end{aligned} \quad (4.5)$$

where

$$E = \frac{\lambda}{r^2G(\phi)}, \quad (4.6)$$

and

$$\lambda = \frac{q}{4\pi}, \quad (4.7)$$

here, we have set $\varepsilon_0 = 1$ for simplicity. The main task is to solve the above equations. Using the classical tachyon potential

$$V(\phi) = \frac{\rho}{4}[(\alpha\phi)^2 - a^2]^2, \quad (4.8)$$

which agrees with all the potential restrictions listed in Eq.(3.19). These conditions are required to guarantee the stabilization of an asymptotically free systems and their associated vacua [319, 318].

Generally, tachyon fields are known to be unstable with negative square masses and velocities greater than the speed of light. Accordingly, tachyons are considered as theoretical particles like the magnetic monopoles. They are presumed to be interacting with each other or other particles at the higher orders in superstring theory to form *tachyon condensation* [340, 38] which is relatively stable. Also, tachyons are known to be unstable fields in quantum field theory (QFT), so it will be necessary to perturb the system about its vacuum $\phi_0 = \pm a/\alpha$ to facilitate its stability [340, 341, 336]. Consequently, there are laid down procedure for removing the tachyonic modes in QFT using perturbation. Since the potential has its minimum at $V(\phi)|_{\phi_0}$, a small disturbance η around the vacuum ϕ_0 will lead to a particle creation with a positive square mass, $V''(\phi)|_{\phi_0} = m_\phi^2$. This will slow down the velocity of the tachyons appreciably, rendering them quite stable and suitable for our analysis. Therefore, we will perturb the field as, $\phi(r) \rightarrow \phi_0 + \eta(r)$ where η is a small disturbance around ϕ_0 which will be referred to as the *glueball field* thus,

$$\begin{aligned} V(\eta) = G(\eta) &= V(\phi)|_{\phi_0} + V'(\phi)|_{\phi_0}\eta(r) + \frac{1}{2}V''(\phi)|_{\phi_0}\eta^2(r) \rightarrow \\ &= \rho a^2 \alpha^2 \eta^2 = \frac{1}{2}m_\phi^2 \eta^2; \end{aligned} \quad (4.9)$$

where $V''(\phi)|_{\phi_0} = m_\phi^2$ is the bosonic mass representing the *glueball mass*. Also, prime and double prime represent first and second order derivatives. Developing the Laplacian in Eq.(4.5),

$$\begin{aligned} \nabla^2(\eta) &= \frac{\partial V}{\partial \phi}|_{\phi_0} + \frac{\partial^2 V}{\partial \phi^2}|_{\phi_0}\eta \rightarrow \\ \nabla^2\eta &= \frac{\partial^2 V}{\partial \phi^2}|_{\phi_0}\eta. \end{aligned} \quad (4.10)$$

Therefore,

$$\eta''(r) + \frac{2}{r}\eta'(r) - m_\phi^2\eta = 0, \quad (4.11)$$

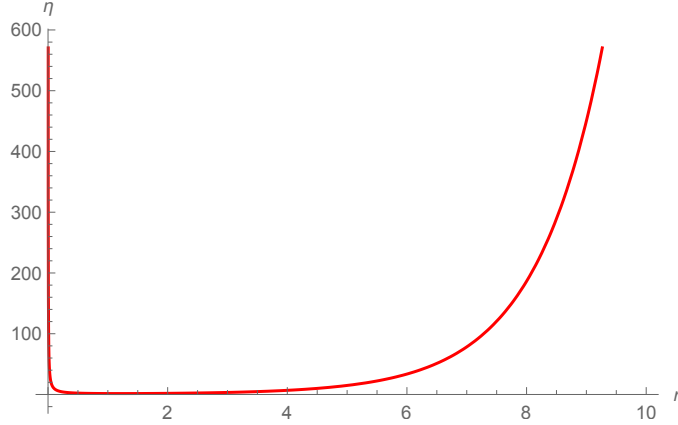
this equation have solutions

$$\eta(r) = \frac{a \cosh(m_\phi r)}{\alpha m_\phi r} \quad \text{and} \quad \eta(r) = \frac{a \sin(m_\phi r)}{\alpha m_\phi r}, \quad (4.12)$$

each solution is suitable for studying the characteristics of the particles in a particular regime i.e. the infrared (IR) and the ultraviolet (UV) regimes respectively. In expressing

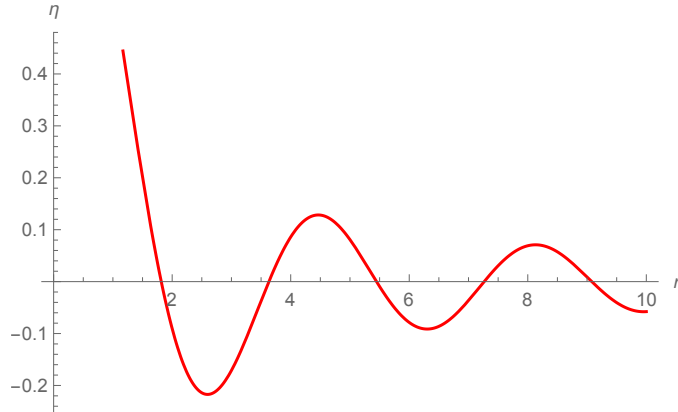
Eq.(4.10) we assume that the particles are far from each other such that $1/r^4$ is negligible, so $\lambda^2 = 0$.

Figure 4.1: The solution of the glueball field in the IR regime.



For the sake of this graph we set $m_\phi = 1$ and $a/\alpha = 1$ to show the characteristics of the solution. The depth of the curve increases with increasing distance inconformity with glueball condensation at low energies.

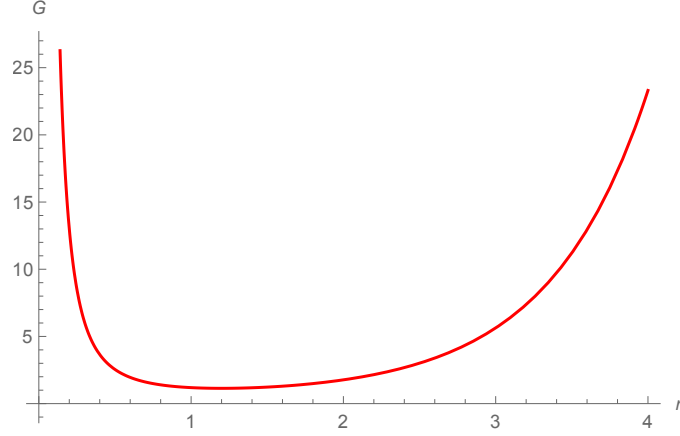
Figure 4.2: The solution of the glueball fields in the UV regime.



In this regime, $m_\phi = 1.73$, $a/\alpha = 1$ and the solution shows an oscillation with increasing distance, representing instability.

The tachyon decay scale α is associated with the decay constant f_α as $f_\alpha = 1/\alpha$. As outlined in Eq.(2.87), thachyon condensations are more favourable in the regimes where the particles are in a confine state i.e. in the low energy regimes. Hence, we have more glueballs condensing at $G(r) \rightarrow 0$ and gradually becomes free at $G(r) \rightarrow 1$. Based on these conditions, we will analyse the glueball condensation using the IR solution in Eq.(4.12) into Eq.(4.9).

Figure 4.3: The characteristics of the glueball potential with distance.



Glueball condensation increases with increasing depth of the curve, this translates into confinement. However, the maximum depth occur at $G(r_*)$ beyond this threshold the curve start rising to the right, this represents hadronization.

4.2 Confining Potentials

Substituting the result on the left side of Eq.(4.12) into the electric field equation yields,

$$\begin{aligned} E &= \frac{\lambda}{r^2 G} \\ &= \frac{\lambda}{\frac{r^2}{2} \left[\frac{a^2 \cosh^2(m_\phi r)}{\alpha^2 r^2} \right]}. \end{aligned} \quad (4.13)$$

Using the well known expression for determining the electrodynamic potential

$$V(r) = \mp \int E dr, \quad (4.14)$$

to determine the confinement potential $V_c(r)$. We get

$$\begin{aligned} V_c(r) &= \frac{2\lambda\alpha^2 \tanh(m_\phi r)}{a^2 m_\phi} + c \\ &= m_\phi \lambda \tanh(m_\phi r), \quad \text{for} \quad a^4 \rho = 1, \end{aligned} \quad (4.15)$$

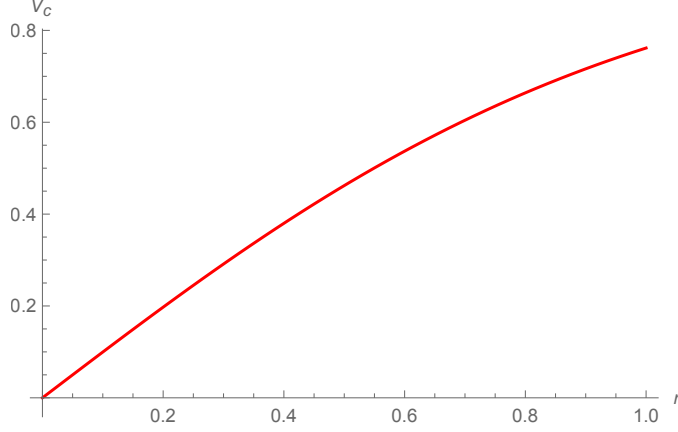
with string tension

$$\sigma_L = m_\phi^2, \quad (4.16)$$

here, we have set the integration constant $c = 0$, $\lambda = 1$ and also used the positive part of potential (4.14). This result gives a linear growth with r representing confinement, at

some distance r_* the potential begins to become flat representing hadronization. From Eq.(4.15), the critical distance r_* within which hadronization is observed can be deduced as $r_* = 1/m_\phi = 1/\sqrt{\sigma_L}$, besides, in flux tube models $r \gg r_*$ [342].

Figure 4.4: The characteristic of the IR potential with distance.



The graph rises linearly, similar to the *flux tube picture* of confinement. However, it flattens up when the separation distance r is increased beyond a certain threshold r_* representing hadronisation.

On the other hand, in the UV regime, we use the solution on the right side of Eq.(4.12) and follow similar procedure as used to determine the confining potential in the IR regime. So,

$$\begin{aligned}
 V_s(r) &= -\frac{2\lambda\alpha^2 \cot(m_\phi r)}{a^2 m_\phi} + c \\
 &\simeq -\frac{2\lambda\alpha^2}{a^2 m_\phi} \left[\frac{1}{m_\phi r} - \frac{m_\phi r}{3} - \mathcal{O}(r^3) \right] + c \\
 &\simeq \frac{2\lambda\rho\alpha^2 a^2}{\rho a^4 m_\phi^2} \left[-\frac{1}{r} + \frac{m_\phi^2 r}{3} \right].
 \end{aligned} \tag{4.17}$$

Thus, for $\lambda = 1$, $\rho a^4 = 1$ and $c = 0$ we obtain the Cornell-like potential

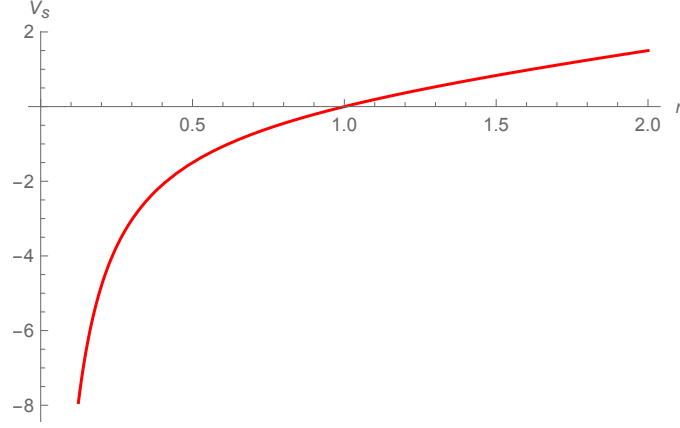
$$V_s(r) = -\frac{1}{r} + \frac{m_\phi^2 r}{3}, \tag{4.18}$$

with string tension

$$\sigma_s = \frac{m_\phi^2}{3}. \tag{4.19}$$

The critical separation distance here will be $r_{*s} = \sqrt{3}/m_\phi = 1/\sqrt{\sigma_s}$, here the critical distance is the minimum distance within which confinement is observed.

Figure 4.5: The behaviour of the confining potential in UV regime with distance.



The particles are asymptotically free until the separation distance is increased beyond 1 fm where confinement begins to set in.

The linear and the Cornell type potentials for color confinement are well known in QCD theory. The linear potential has been widely studied by various groups [56, 161, 172, 343, 344, 345, 346, 347], and the results gives a good agreement with experimental data for quarkonium spectroscopy and its decay characteristics. In the model framework, m_ϕ^2 is the glueball mass representing the confining strength. This kind of potential fails to describe the partonic structure of the color fields as observed in deep-inelastic scattering experiments. Accordingly, linear confining potentials only gives a good description of the particles in the IR regime [167, 168].

On the other hand, Cornell potential (4.18) is known for confining heavy quarks, it is motivated by lattice QCD (LQCD). It carries the main properties of the QCD theory. The asymptotically freedom nature of the theory at $r \rightarrow 0$ is depicted here by the Coulomb-like part of the potential due to one-gluon exchange. Besides, the *color confinement* behaviour shown by the QCD theory at low energy regimes $r \rightarrow \infty$, is also depicted by the linear part of the potential as illustrated in Fig. 4.5. This type of potentials are motivated by QCD lattice calculations. The strength of the Coulomb part is 1 and the confining strength is $m_\phi^2/3$. Taking the string tension for a hadron of mass 1 GeV and radius 1 fm as estimated through electron scattering to be $\sigma_s \sim 1\text{GeV/fm}$. Using this string tension as the benchmark, the *glueball mass* can be determine as $m_\phi = 1.73\text{ GeV}$ in the UV regime. This is precisely the same as the *scalar glueball mass* with quantum number $J^{PC} = 0^{++}$ determined through QCD lattice simulations 1.7 GeV [29, 348, 349, 350, 351, 352]. For recent developments see Refs.[353, 354]. By this result, we have two different glueball

masses one in the UV regime and the other in the IR regime, $m_\phi = 1 \text{ GeV}$, where the gluons are already known to be in a confined state. Thus, higher gluon bound states are required in the UV regime than in the IR regime in order to obtain *color confinement*, if we assume that $\sigma_L = \sigma_s \sim 1 \text{ GeV/fm}$.

Moreover, from the afore discussions we can deduce that the string tension is related to the glueball mass as $m(0^{++})/\sqrt{\sigma} = \text{constant}$. Its magnitude depends on the approach and the motivation of the investigator. In this model framework, this ratio will have two different values depending on the regime under consideration. In the UV regime, we have $m_\phi/\sqrt{\sigma_s} = 1.73$ and in the IR regime we get $m_\phi/\sqrt{\sigma_L} = 1$. This ratio is one of the exactly known physical quantities in QCD, because its correction is the leading order of $\mathcal{O}(1/N_c)$ in the $SU(\infty)$ limit. This quantity have been determined using different approaches, lattice QCD calculations, QCD sum rule, flux tube model and the constituent glue model and its value is precisely known [355, 356, 357].

4.3 (De)confinement Transition in Tachyon Matter at Finite Temperature

In this section we intend to elaborate on the net confining potentials in both the IR and the UV regimes as a function of temperature T and their corresponding string tensions. This phenomenon is generally referred to as the *thermal QCD*. Indeed, proper understanding of this subject is particularly important in understanding the early universe, the dynamics of compact astrophysical objects such as the inner core of neutron stars and recent observations in the heavy ion collision processes in RHIC [358] and LHC [359]. We will look at the phase transitions from confining phase at $T < T_c$, deconfining and QGP phases at $T \geq T_c$, where quarks and gluons behave in a random manner due to asymptotic free behaviour of the particles. Despite some QCD lattice simulation results pointing to confinement beyond the critical temperature $T > T_c$ [360], in simple models such as the one intended for this thesis, confinement can be obtained at $T < T_c$, deconfinement at $T \geq T_c$ and QGP phase at $T > T_c$. Different approaches have been used in investigating this subject, common among others are the Wilson and Polyakov loop corrections [110, 111, 112, 113]. However, this subject remain opened due to lack of consensus in the order of the temperature correction to the associated string tension σ

and consolidation of the various methods. The projected characteristics for σ are that, it decreases sharply with temperature $T > T_c$, breaks at $T = T_c$ and continue decreasing slowly if the temperature keeps increasing beyond T_c i.e. $T_c > T$ [2].

We present a temperature correction in the tachyon field ϕ which goes to correct the temperature in both the potential and the color dielectric function as we have already established that $V(\phi) = G(\phi)$. That notwithstanding, we will shift from the usual way of treating field theory where the perturbation is taken about the vacuum $\langle\phi\rangle_0 = 0$ to include the particle densities, their interactions and their thermal distributions about the vacuum. Nonetheless, we will introduce a thermal shift such that $\phi \rightarrow \phi + \phi_T$, following the condition $\langle\phi_T\rangle = 0$. Explicitly computing $\langle\phi_T^2\rangle$, we consider a shift in the vacuum such that $\phi \rightarrow \phi + \phi_0$, so the potential becomes

$$\begin{aligned} V(\phi) = G(\phi) &= V(\phi)|_{\phi_0} + V'(\phi)|_{\phi_0}\phi + \frac{1}{2}V''(\phi)|_{\phi_0}\phi^2 \\ &= \frac{1}{2}m_\phi^2\phi^2. \end{aligned} \quad (4.20)$$

As a result, the Lagrangian density (4.1) then becomes

$$\tau_p^{-1}\mathcal{L} = -\frac{1}{2}\phi\left[\square + m_\phi^2\right]\phi - \frac{1}{4}G(\phi)F_{\mu\nu}F^{\mu\nu}. \quad (4.21)$$

The Feynman propagator for this Lagrangian for the scalar sector will be

$$D_\phi(p) = \frac{i}{p^2 - m_\phi^2 + i\varepsilon}; \quad (4.22)$$

where $p_\mu \rightarrow i\partial_\mu$ and ε is an infinitesimal constant that regulates the divergence at $p^2 = m_\phi^2$. Accordingly, the Green's function of the propagator reads,

$$G_F(x - x') = \langle 0|T[\phi(x)\phi(x')]|0\rangle = \frac{i}{(2\pi)^4} \int \frac{e^{-ip(x-x')}d^4p}{p^2 - m_\phi^2 + i\varepsilon}; \quad (4.23)$$

here, T is the time ordering operator. We set $x = x'$ to eliminate the time ordering operator consequently,

$$\begin{aligned} G_F(0) &= \langle 0|\phi^2(x)|0\rangle = \frac{i}{(2\pi)^4} \int \frac{d^4p}{p^2 - m_\phi^2 + i\varepsilon} \\ &= i \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{dp_0}{2\pi(p^2 - m_\phi^2 + i\varepsilon)} \\ &= - \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{dp_0}{4\pi(\mathbf{p}^2 + m_\phi^2)}; \end{aligned} \quad (4.24)$$

where p_0 is the time component of the four momentum and $\mathbf{p}$ is the spatial component. We Wick rotate such that $p_0 \rightarrow -ip_0$ to remove the divergence at $p^2 = m_\phi^2$, so we can

drop ε and proceed safely. Accordingly, $p^2 - m_\phi^2 = -2(\mathbf{p}^2 + m_\phi^2)$ with $p_0^2 \rightarrow (-ip_0)^2 = -(\mathbf{p}^2 + m_\phi^2)$ giving rise to

$$\begin{aligned} \int \frac{dp_0}{4\pi(\mathbf{p}^2 + m_\phi^2)} &= \int \frac{\mathbf{p}d\mathbf{p}}{2(\mathbf{p}^2 + m_\phi^2)^{3/2}} \\ &= - \int \frac{d(\mathbf{p}^2 + m_\phi^2)^{-1/2}}{2} \\ &= - \frac{1}{2\sqrt{\mathbf{p}^2 + m_\phi^2}}. \end{aligned} \quad (4.25)$$

Therefore, Eq.(4.24) becomes

$$\langle 0|\phi^2|0\rangle = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\sqrt{\mathbf{p}^2 + m_\phi^2}}; \quad (4.26)$$

where $E_\phi = \sqrt{\mathbf{p}^2 + m_\phi^2}$ is the energy of a single particle with momentum $\mathbf{p}$ and mass m_ϕ . We will now generalize the expectation value of ϕ^2 , not limiting it to the vacuum, thus $\langle 0|\phi^2|0\rangle \equiv \langle \phi^2\rangle$. We split ϕ into creation and annihilation parts $\phi = \phi^\dagger + \phi^-$ and apply the standard scalar field quantization procedure in terms of creation $a_{\mathbf{q}}^\dagger$ and annihilation $a_{\mathbf{p}}^-$ operators, where $\mathbf{q}$ and $\mathbf{p}$ are their momenta respectively. The scalar field operators are related to the ladder operators through the commutation relation

$$[\phi_{\mathbf{p}}^-, \phi_{\mathbf{q}}^\dagger] \equiv [a_{\mathbf{p}}^-, a_{\mathbf{q}}^\dagger] = 2p_0\delta^3(\mathbf{p} - \mathbf{q}). \quad (4.27)$$

Supposing that $a_{\mathbf{q}}^\dagger$ creates a particle of momentum $\mathbf{q}$ then $a_{\mathbf{p}}^-$ will destroy a particle of momentum $\mathbf{p}$. Hence, we can employ the identities $[a_{\mathbf{p}}^-, a_{\mathbf{p}}^\dagger] = 1$ and $\langle a_{\mathbf{p}}^\dagger a_{\mathbf{p}}^- \rangle = n_p$. Thus, the expression in Eq.(4.26) yields,

$$\begin{aligned} \langle \phi^2 \rangle &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_\phi} \langle (a_{\mathbf{p}}^- + a_{\mathbf{p}}^\dagger)^2 \rangle \\ &= \frac{1}{(2\pi)^3} \int_0^\infty \frac{d^3\mathbf{p}}{E_\phi} \left(\frac{1}{2} + n_p \right). \end{aligned} \quad (4.28)$$

Subtracting the fundamental scalar field propagator $\langle \phi^2 \rangle_0$ from Eq.(4.28) gives

$$\langle \phi^2 \rangle = \frac{1}{(2\pi)^3} \int_0^\infty \frac{d^3\mathbf{p}}{E_\phi} n_p; \quad (4.29)$$

where n_p is the particle number density [361, 362]. For Bose gas or Bose-Einstein condensates, we can replace n_p with

$$n_p \equiv n_B(E_\phi) = \frac{1}{e^{(\sqrt{p^2 + m_\phi^2})/T} - 1}; \quad (4.30)$$

where T is temperature. Consequently, we can express Eq.(4.29) in terms of $n_B(E_\phi)$ as

$$\langle \phi_T^2 \rangle = \frac{1}{(2\pi)^3} \int_0^\infty \frac{d^3\mathbf{p}}{E_\phi} n_B(E_\phi); \quad (4.31)$$

where $n_B(x) = (e^{x\beta} - 1)^{-1}$ is the Bose-Einstein distribution. Assuming a high energy limit, we can approximate $kT \gg m_\phi c^2$ and $E_\phi \approx pc$ where k is the Boltzmann constant and c is the speed of light in vacuum [123] hence,

$$\langle \phi_T^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty \frac{p^2 dp}{E_\phi (e^{E_\phi \beta} - 1)} \simeq \frac{T^2}{12}; \quad (4.32)$$

where $\beta = 1/T$.

4.3.1 Tachyon Potential at Finite Temperature

Introducing the temperature into the potential using the transformation $\phi \rightarrow \phi + \phi_T$, we obtain

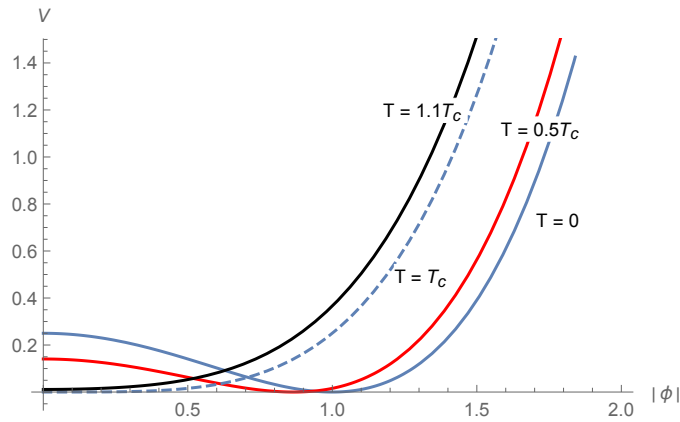
$$\begin{aligned} V(\phi, T) &= \frac{\rho}{4} \left[\alpha^2 (\phi + \phi_T)^2 - a^2 \right]^2 \\ &= \frac{\rho}{4} \left[(\alpha\phi)^2 + \frac{\alpha^2 T^2}{12} - a \right]^2 \\ &= \frac{\rho}{4} \left[(\alpha\phi)^2 - \tilde{a}^2 \right]^2; \end{aligned} \quad (4.33)$$

here, we have set the cross terms to zero because $\langle \phi_T \rangle = 0$,

$$\tilde{a}^2 = a^2 \left(1 - \frac{T^2}{T_c^2} \right) \quad \text{and} \quad T_c = \frac{2a\sqrt{3}}{\alpha}, \quad (4.34)$$

where T_c is the critical temperature.

Figure 4.6: A graph of $V(|\phi|, T)$ against $|\phi|$.



It shows a first order phase transition at $T = T_c$.

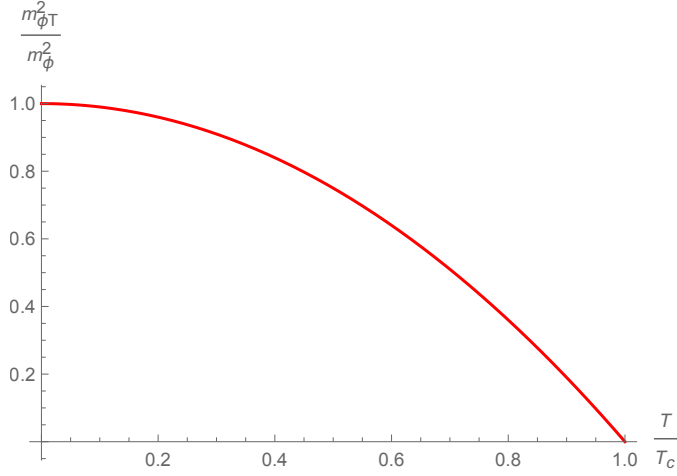
The vacuum of the potential can now be expressed as $\phi_0 = \tilde{a}/\alpha$ rendering it temperature dependant. Also, the thermal fluctuating glueball mass can be expressed as

$$\begin{aligned} m_\phi^2(T) &= \frac{\partial^2 V(\phi, T)}{\partial \phi^2} \Big|_{\phi_0} = 2\rho\alpha^2\tilde{a}^2 \\ &= m_\phi^2 \left[1 - \frac{T^2}{T_c^2} \right], \end{aligned} \quad (4.35)$$

consequently, Eq.(4.9) becomes

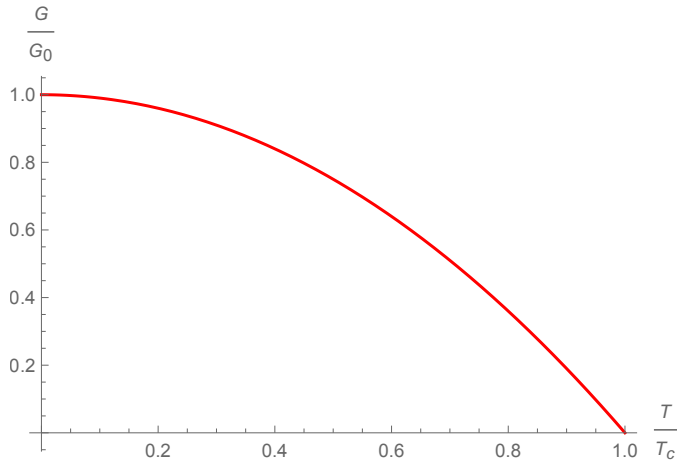
$$V(\eta, T) = G(\eta, T) = \frac{1}{2}m_\phi^2(T)\eta^2 = G(\eta) \left[1 - \frac{T^2}{T_c^2} \right]. \quad (4.36)$$

Figure 4.7: Thermal fluctuating glueball mass.



The glueball mass decreases with temperature and vanishes at $T = T_c$. This also represents the behaviour of the string tension because they are related as $m_\phi^2(T) = \sigma_L(T)$.

Figure 4.8: Thermal fluctuating glueball potential $G(\eta, T)/G(\eta)$.



Increasing in temperature decreases the glueball condensation until it vanishes at $T = T_c$. Thus, glueballs are more likely to condense at low temperatures than it may at higher temperatures.

Also, the solutions in Eq.(4.12) can be expressed in terms of T by substituting the glueball mass m_ϕ with the thermal fluctuating glueball mass $m_\phi(T)$.

4.3.2 Confining Potentials at a Finite Temperature

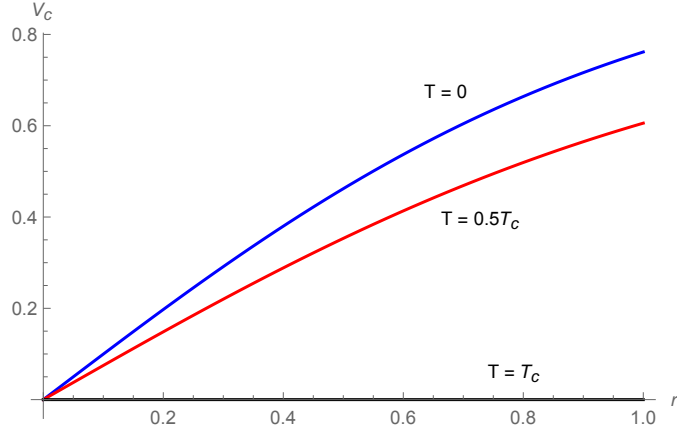
From the afore analysis, the confining potential (4.15) can be written in terms of the thermal fluctuating glueball mass as

$$V_c(r, T) = m_\phi(T)\lambda \tanh[m_\phi(T)r], \quad (4.37)$$

with string tension

$$\sigma_L(T) = m_\phi^2(T). \quad (4.38)$$

Figure 4.9: The characteristics of the thermal fluctuating vector potential with distance.



The gradient of the graph decreases with increasing T from $T = 0$ to $T = 0.5T_c$. At $T = T_c$ the potential vanishes and the particles states to hadronize.

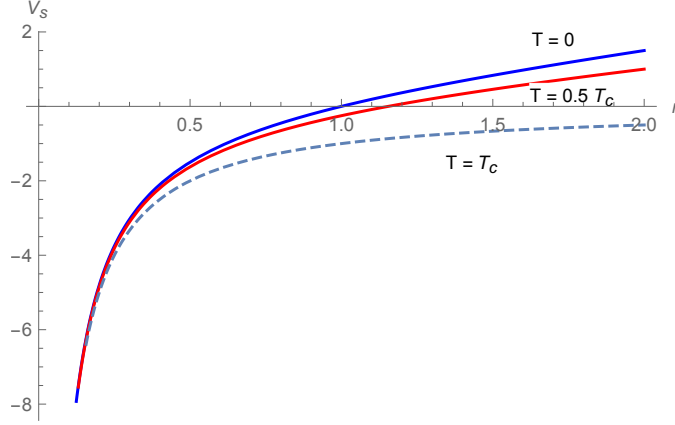
Temperature influences confinement such that at $T = 0$ we have strong confinement as shown in Figs. 4.4 and 4.9, as the temperature is increased, the particles acquire more energy and become asymptotically free at $T = T_c$ leading to a break in the string tension $\sigma(T) = 0$ giving rise to hadronization. On the other hand, the potential in the UV regime can also be expressed by substituting the glueball mass into Eq.(4.18) with the thermal fluctuating glueball mass, so

$$V_s(r, T) = -\frac{1}{r} + \frac{m_\phi^2(T)r}{3}, \quad (4.39)$$

with string tension

$$\sigma_s(T) = \frac{m_\phi^2(T)}{3}. \quad (4.40)$$

Figure 4.10: Thermal fluctuating UV potential.



Increase in temperature reduces the strength of confinement from $T = 0$ to $T = 0.5T_c$ and at $T = T_c$ the particles become asymptotically free representing thermal deconfinement phase.

4.4 QCD-Like Vacuum

4.4.1 Gluodynamics

Gluodynamics for SU(3) color group is governed by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}, \quad (4.41)$$

which is invariant under conformal transformation $x \rightarrow \lambda x$ in the classical limit. This gives rise to an associated scale current s_μ which is related to the trace of the energy momentum tensor as

$$\partial^\mu s_\mu = \theta_\mu^\mu(x). \quad (4.42)$$

In the absence of quantum corrections to Eq.(4.41) $\theta_\mu^\mu(x) = 0$, and the conformal invariance persists. This leads to vanishing gluon condensate $\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle = 0$. On the contrary, when quantum corrections are considered, the Lagrangian density becomes

$$\mathcal{L}' = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + |\epsilon_v|; \quad (4.43)$$

where $-|\epsilon_v|$ is the vacuum energy density which serves as quantum correction and breaks the conformal symmetry [1, 2]. However, in the classical limit $|\epsilon_v| \rightarrow 0$ we recover Eq.(4.41). The quantum correction leads to non-vanishing gluon condensate $\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle \neq 0$ [363] and QCD energy momentum trace anomaly

$$\theta_\mu^\mu = \frac{\beta(g)}{2g} F_{\mu\nu}^a F^{a\mu\nu}, \quad (4.44)$$

with corresponding vacuum expectation value,

$$\langle \theta_\mu^\mu \rangle = -4|\epsilon_v|. \quad (4.45)$$

Where

$$\beta(g) = -\frac{11g^3}{(4\pi)^2}; \quad (4.46)$$

is the leading term of the β -function expansion (2.13). These properties are consequences of the broken down conformal symmetry [364, 365].

4.4.2 Gluon Condensation

We will calculate the trace of energy momentum tensor (θ_μ^μ) from the Lagrangian density (4.1) using the relation [1]

$$\theta_\mu^\mu = 4V(\eta) + \eta \square \eta. \quad (4.47)$$

The potential term in the Lagrangian clearly breaks the conformal symmetry, giving rise to non-zero gluon condensate [318]. Substituting the equations of motion (4.2) into (4.47) yields,

$$\begin{aligned} \theta_\mu^\mu &= 4V(\eta) - \eta \frac{\partial G}{\partial \eta} F^{\mu\nu} F_{\mu\nu} - \eta \frac{\partial V}{\partial \eta} \\ &= 4\tilde{V} - \eta G'(\eta) F^{\mu\nu} F_{\mu\nu}; \end{aligned} \quad (4.48)$$

where

$$\tilde{V}(\eta) = V(\eta) - \frac{\eta}{4} \frac{\partial V}{\partial \eta}. \quad (4.49)$$

Rescaling $\tilde{V}(\eta)$ with the vacuum energy density $-|\epsilon_v|$ [318, 2], i.e.,

$$\tilde{V}(\eta) \rightarrow -|\epsilon_v| \tilde{V}(\eta), \quad (4.50)$$

and using this result together with the expectation value of θ_μ^μ i.e. Eqs.(4.45) and (4.48) yields,

$$\langle \eta \tilde{G}'(\eta) F^{\mu\nu} F_{\mu\nu} \rangle = 4|\epsilon_v| \langle 1 - \tilde{V}(\eta) \rangle. \quad (4.51)$$

Using the glueball field η defined in Sec. 4.1, we can express

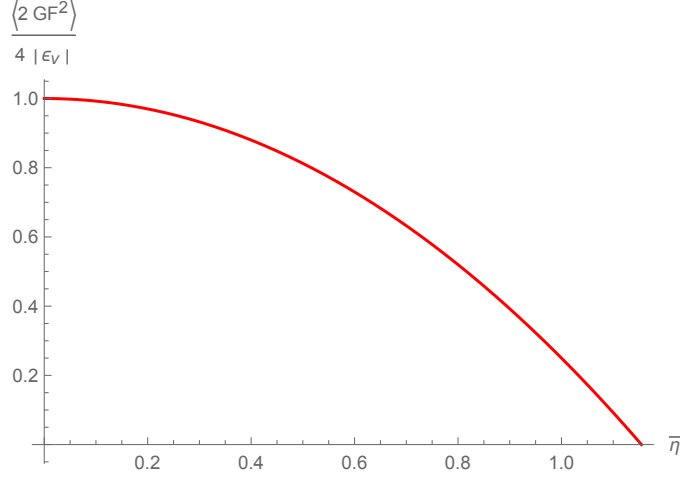
$$\tilde{V}(\eta) = \frac{1}{4} m_\phi^2 \eta^2 \quad \text{and} \quad \eta G'(\eta) = m_\phi^2 \eta^2 = 2G(\eta). \quad (4.52)$$

Accordingly,

$$\langle 2G(\eta)F^{\mu\nu}F_{\mu\nu} \rangle = 4|\epsilon_v| \left\langle 1 - \frac{m_\phi^2 \eta^2}{4} \right\rangle, \quad (4.53)$$

we retrieve the classical result when we set $|\epsilon_v| \rightarrow 0$.

Figure 4.11: Gluon condensation with mean glueball field.



The glueball fields have two major solutions, one at $\bar{\eta} = 0$ giving rise to maximum gluon condensation and the other at $\bar{\eta} \simeq 1$ leading to vanishing condensate. Thus, the gluon condensate decreases with increasing mean glueball field.

Since the glueball field η transforms as $\phi \rightarrow \eta + \phi_0$, similar to the transformation leading to the quantization of the tachyon field ϕ , i.e. $\phi \rightarrow \phi + \phi_0$, elaborated in Sec. 4.3. Quantization of η takes a similar form as Eq.(4.28) i.e.,

$$\langle \eta^2 \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3\mathbf{p}}{E_\eta} \left(\frac{1}{2} + n_p \right); \quad (4.54)$$

where $E_\eta = \sqrt{\mathbf{p}^2 + m_\phi^2}$ is the energy of a single valence gluon. Therefore, the condensate as represented in Eq.(4.53) consists of creation and annihilation of particles in a haphazard manner in the vacuum. Assuming high temperature limits and a Bose-Einstein condensate similar to Eq.(4.30), Eq.(4.54) can be approximated as

$$\langle \eta_T^2 \rangle \simeq \frac{T^2}{12}, \quad (4.55)$$

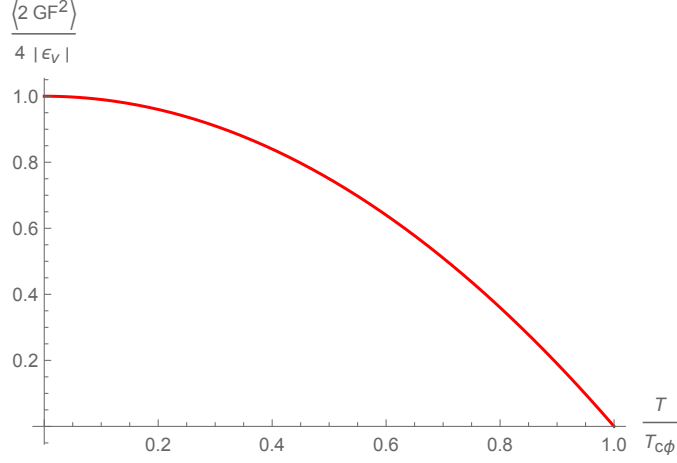
and Eq.(4.53) reads

$$\langle 2G(\eta)F^{\mu\nu}F_{\mu\nu} \rangle = 4|\epsilon_v| \left[1 - \frac{T^2}{T_{c\phi}^2} \right]; \quad (4.56)$$

where

$$T_{c\phi} = \frac{4\sqrt{3}}{m_\phi}. \quad (4.57)$$

Figure 4.12: Changes in gluon condensation with temperature.



The maximum condensate occur at $T = 0$ and it vanishes at the critical temperature $T = T_{c\phi}$.

So, the condensate decreases with temperature increase until it vanishes at $T = T_{c\phi}$. On the other hand, temperature fluctuations in the condensate can also be studied by considering the temperature fluctuations in the glueball mass (4.35) and the glueball field. Consequently,

$$\begin{aligned}
 \langle 2G(\eta)F^{\mu\nu}F_{\mu\nu} \rangle &= 4|\epsilon_v| \left[1 - \frac{m_\phi^2}{4} \left[1 - \frac{T^2}{T_c^2} \right] \left(\frac{T^2}{12} \right) \right] \\
 &= 4|\epsilon_v| \left[1 - \frac{m_\phi^2}{48} T^2 + \frac{m_\phi^2}{48} \frac{T^4}{T_c^2} \right] \\
 &= 4|\epsilon_v| \left[1 - \frac{T^2}{T_{c\phi}^2} + \frac{T^4}{T_{c\phi 1}^4} \right], \tag{4.58}
 \end{aligned}$$

in the last step, we substituted

$$T_{c\phi 1}^4 = T_{c\phi}^2 T_c^2 = \left(\frac{48}{m_\phi^2} \right) \left(\frac{12a^2}{\alpha^2} \right) = \frac{576a^2}{m_\phi^2 \alpha^2}, \tag{4.59}$$

hence, $T_{c\phi 1}^4 \gg T_{c\phi}^2$. In this case, we obtain the maximum at $T = 0$, and the minimum at $T = T_{c\phi}$.

4.4.3 Strong Running Coupling and β -Function

Comparing Eqs.(4.44) and (4.48) leads to

$$\begin{aligned}
 \frac{\beta(g)}{2g} &= -\eta G'(\eta) \rightarrow \\
 \beta(1/r^2) &= 2g\eta G'(\eta). \tag{4.60}
 \end{aligned}$$

Using the well-known relation for calculating the QCD β -function from renormalization group theory [191]

$$\beta(Q^2) = Q^2 \frac{d\alpha_s}{dQ^2}, \quad (4.61)$$

we can deduce

$$\beta(\eta^2) \simeq -\eta \frac{d(G)}{d\eta} = -m_\phi^2 \eta^2(r) = \beta(1/r^2) \quad \text{setting} \quad g = 1. \quad (4.62)$$

From these expressions, we can extract the strong running coupling to be $\alpha_s(\eta^2) = G(\eta) = \alpha_s(1/r^2)$. The negative β -function reflects the asymptotic free nature of the QCD theory. It also points to the anti-screening nature of the theory at higher energy regimes. Additionally, the running coupling carries information on the growing precision of hadron scattering experiments at higher momentum limits. At low momentum regimes within the size of the hadron mass, it provides an insight into the hadron structure, color confinement and hadronization. Since the UV solution obtained above provides an insight into both confinement (linearly growing part) and the deconfining part (the Colombian part) potentials. We will adopt it for the analysis in this subsection thus,

$$\alpha(1/r^2) = \left[1 - \frac{m_\phi^2 r^2}{3} \right]. \quad (4.63)$$

Hence, noting that $Q \equiv 1/r$ is the spacelike momentum of the gluons, we can express

$$\alpha(Q^2) = \left[1 - \frac{m_\phi^2}{3Q^2} \right]. \quad (4.64)$$

This equation can be generalized when we express the spacelike momentum in terms of the four vector momentum $Q^2 \equiv -q^2$ thus,

$$\alpha(q^2) = \left[1 + \frac{m_\phi^2}{3q^2} \right], \quad (4.65)$$

and the β -function on the other hand becomes,

$$\beta(q^2) = -2 \left[1 + \frac{m_\phi^2}{3q^2} \right]. \quad (4.66)$$

There is a 'so called' Landau ghost pole at $q^2 \rightarrow 0$ in both the β -function and the strong running coupling. The singularity signifies the end of perturbation QCD theory and the beginning of nonperturbation QCD. This is generally attributed to the dynamically generated gluon mass and its self-interactions with hadron degrees of freedom, leading

to color confinement [254]. Accordingly, there is a dynamically generated gluon mass $q^2 \cong m_A^2$ [255, 256] at $q^2 \rightarrow 0$ which gradually vanishes at $q^2 \rightarrow \infty$. In this regard, the singularity can be removed by adding m_A^2 to q^2 to serve as the freezing point [255, 257] giving a regular behaviour at $q^2 \rightarrow 0$, consequently,

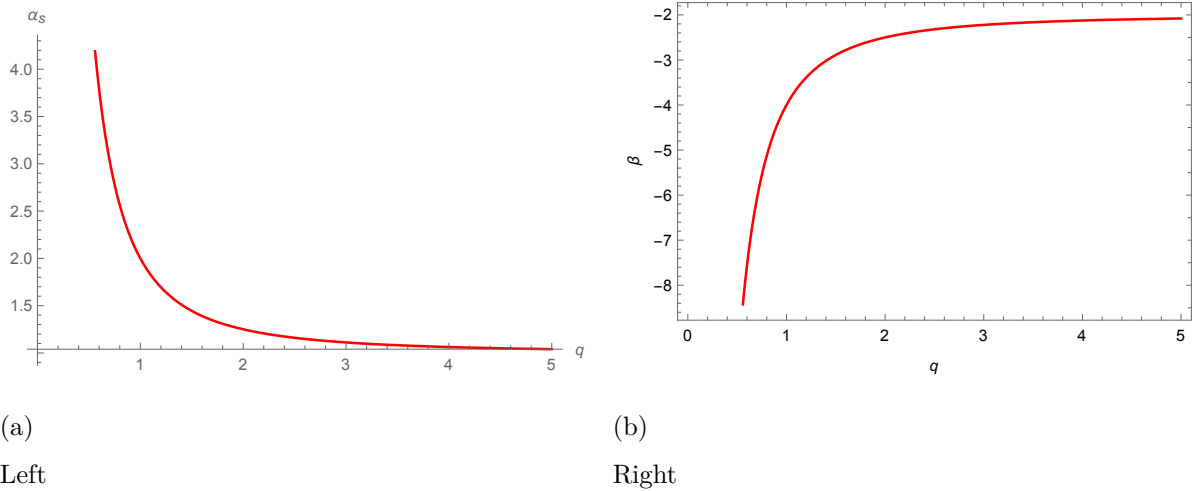
$$\alpha(q^2) = \left[1 + \frac{m_\phi^2}{3(q^2 + m_A^2)} \right], \quad (4.67)$$

and

$$\beta(q^2) = -2 \left[1 + \frac{m_\phi^2}{3(q^2 + m_A^2)} \right]. \quad (4.68)$$

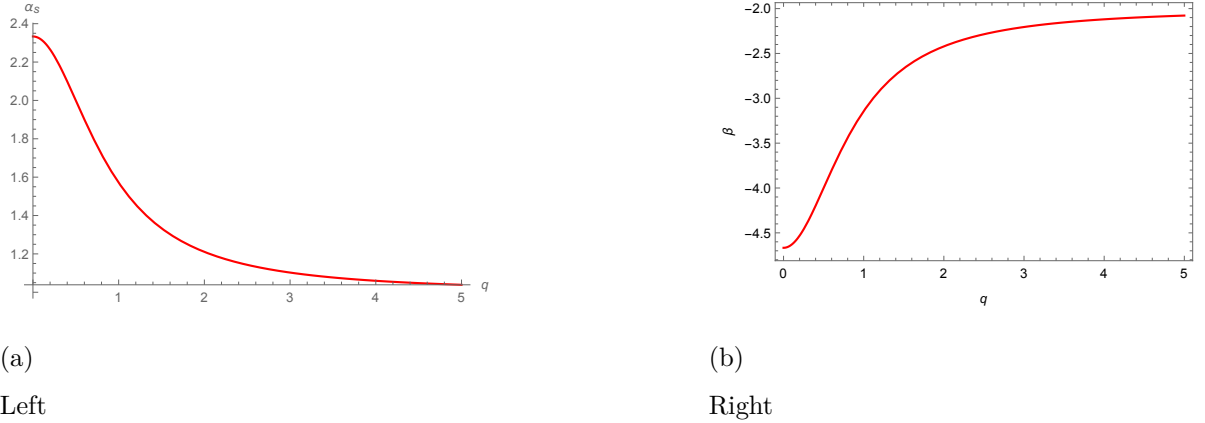
By these relations, the effect of the singularity is completely removed leading to a well defined freezing point at $q^2 \rightarrow 0$. The effect of the gluon mass is negligible at higher energy regimes and more pronounced at lower energies [189].

Figure 4.13: Strong Running Couple α_s (left) and β -Function (right) against q , with a Landau Ghost Pole.



The graphs show an unphysical behaviour at $q \rightarrow 0$, this is due to the presence of dynamically generated gluon mass. The self-interacting gluons and the strong force that exist between them is capable of creating bound states with hadron degrees of freedom. These graphs depict the behaviour of α_s and β observed from pQCD.

Figure 4.14: Strong Running Coupling α_s (left) and β -Function (right) against q with a freezing point at $q \rightarrow 0$.



Here, the Landau singularity has been fixed by introducing the gluon mass m_A . The presence of the gluon mass is more pronounced at $q \rightarrow 0$ and gradually vanishes in the limit $q \rightarrow \infty$. Consequently, $\alpha_s(0) \simeq 2.3$ and $\beta(0) \simeq -4.7$ from the graphs.

4.5 Analysis

The objective of this chapter was to determine confinement of gluons at $T = 0$ and at a finite T . Motivated by Abelian dominance in *color confinement* phenomenon, the relationship established between the dielectric function $G(\phi)$ and the classical tachyon potential $V(\phi)$ simplified the calculations. The $G(\phi)$ coupled to the gauge field $F^{\mu\nu}F_{\mu\nu}$ absorbs the long distance dynamics of the photon propagator preventing it from decoupling at longer wavelengths. To that extent, the classically quantized color dielectric function $G(\eta)$ behaves like QCD beta function bringing about non vanishing gluon condensation. Accordingly, the dielectric function facilitates the self interactions that exist among the valance gluons. Also, $G(\eta)$ leads to glueball condensation which translate into color confinement. Thus, higher glueball condensation means stronger confinement and the vice versa [317]. Confinement is realized in low energy regimes i.e. long distance inter particle separation and/or low temperatures.

The model produces *flux tube*-like and *Cornell*-like potentials in the IR and the UV regimes respectively. We introduced temperature into the potential by using field quanta distribution approach. This led to obtaining thermal fluctuating glueball mass which serve as a medium for obtaining thermal fluctuating confining potentials. Because the

string tensions are associated with the potentials ($V_c(r, T)$ and $V_s(r, T)$) and the glueball mass in a simple form, it facilitates the introduction of temperature into the model. The glueball masses generated in the model framework are determined to be $m_\phi = 1 \text{ GeV}$ and $m_\phi = 1.73 \text{ GeV}$ for IR and UV regimes respectively. While the critical distances for IR and UV regimes were calculated to be $r_* = r_{*s} = 1 \text{ fm}$. The critical distance in the IR regime mark the beginning of hadronization whilst the one in the UV regime mark the beginning of confinement.

In conclusion, the model agrees largely with other phenomenological confining models where the strength of the confinement increases with increasing distance. Beyond some critical distance hadronization sets in. The hadronization phase is quite obvious with the IR model where Fig. 4.4 becomes flat with increasing distance. This behaviour is not clearly obvious with the Cornell-like potential Fig. 4.5 in the UV regime. In the linear confining potential the particle starts hadronizing at the critical distance r_* , with the Cornell confining potential on the other, confinement sets in at the critical distance r_{*s} . We obtained confinement at $T < T_c$, thermal deconfinement at $T = T_c$ and QGP phase at $T > T_c$. Several lattice QCD calculations agree with temperature correction in the order of $-T^2$ which is also consistent with the results obtained in this chapter [113, 366, 367, 112].

Chapter 5

Confinement of Fermions

In this chapter we will use Lagrangian density (4.1) and introduce standard model fermion fields [368, 369]. So, we can investigate confinement of fermions with at least one heavy central quark similar to HQET. Thus, we will construct an effective Lagrangian density for the study by coupling Eqs.(3.30) and (4.1) in a suitable manner to describe the fermionic tachyons intended for this chapter. The motivation for this approach are:

- We will be able to compute both scalar and vector confining potentials by considering a heavy quark source surrounded by relatively light quarks in a slow motion.
- We will be able to establish a relationship between the color dielectric function and the tachyon potential to simplify the calculations.
- It is easy to calculate the associated gluon condensate and other IR properties associated with QCD.
- The magnitudes of the 'bare' quark mass and the constituent quark masses can be determined.

Most of these motivations were also the basics for the previous chapters as well.

In a static quark frame, i.e. in the absence of the *chromomagnetic field* we can do the analysis based on the *kinematics Thomas spin-orbit interactions*. These interactions are relativistic corrections in nature, and are applied to both scalar and vector confining potentials [370]. Even though, *Thomas precession* is scalar in nature and suggests the existence of scalar potential, it does not warrant its connection with the QCD theory. However, it coincides with long distance spin-orbit interactions in QCD, i.e. in the low

energy regime of the theory. Vector and scalar potentials occur at short and long range Thomas spin-orbit interactions respectively. Besides, these interactions do not depend on the spin-orbit interactions of the quarks, which is consistent with QCD theory [371, 372]. In spite of the widely successful nature of the scalar potential in describing hadron models phenomenologically, there have not been successful attempt in linking it to QCD theory [373, 374]. Interestingly, it has been shown in the framework of slowly moving light quarks about a central heavy source that there is coexistence of relativistic spin dependent [375, 376] and spin independent [377] corrections. This will naturally lead to vector and scalar confining potentials. This will be one of the objectives of this chapter. Also, we will present an analysis on vector potential dominance by calculating the coupling strengths of both potentials and comparing them.

Finally, the approach here will be similar to the procedure adopted in Chap. 4. That is, we will use an Abelian QED approach to approximate the non-Abelian QCD theory while the color dielectric function is responsible for the strong interaction properties, the associated vacuum [54, 378] and glueball condensation.

5.1 Lagrangian Density for the Study

The study will be centred on the suitable combination of the Lagrangian densities (3.30) and (4.1),

$$\mathcal{L} = -\frac{1}{4}G(\phi)F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \bar{\psi}(i\gamma^\mu\partial_\mu + q\gamma^\mu A_\mu - m_q G(\phi))\psi. \quad (5.1)$$

Here, m_q is the 'bare' quark mass sometimes referred to as the current quark mass for light particles or the running quark mass if the particles under consideration are heavy. It is the same as the mass that appear in the classical QCD Lagrangian [336]. Meanwhile, $M(\phi) = m_q G(\phi)$ will be seen as the 'so called' *constituent quark mass function*. It will be thoroughly discussed in the latter sections of this chapter and its magnitude relative to regimes under consideration determined. Fast forward, we find that this mass function is equivalent to the scalar potential energy $S(r)$ of the model for an arbitrary separation distance r . Additionally, the 'bare' quark mass breaks down the *chiral symmetry explicitly* while $M(r)$ can lead to both *explicit and dynamic chiral symmetry breaking* comparable to the renormalized mass associated with the QCD Lagrangian [79, 379].

Furthermore, the 'bare' mass m_q term together with the gauge field $F^{\mu\nu}F_{\mu\nu}$ that appear in the Lagrangian guarantees that we will have both scalar and vector contributions to the resulting potentials. The scalar potential is spearheaded by the mass and the vector potential is facilitated by the gauge fields. The equations of motion associated with Eq.(5.1) are

$$\partial_\mu \partial^\mu \phi + \frac{1}{4} \frac{\partial G(\phi)}{\partial \phi} F^{\mu\nu} F_{\mu\nu} + \frac{\partial V(\phi)}{\partial \phi} - \bar{\psi} m_q \psi \frac{\partial G(\phi)}{\partial \phi} = 0, \quad (5.2)$$

$$- (i\gamma^\mu \partial_\mu + q\gamma^\mu A_\mu) \psi + m_q G(\phi) \psi = 0, \quad (5.3)$$

and

$$\partial_\mu [G(\phi) F^{\mu\nu}] = \bar{\psi} q \gamma^\nu \psi. \quad (5.4)$$

Using similar definition of indices as used in previous chapter to ensure that only static sector of the scalar field is available for analysis. We also choose $j^\nu = (\rho, \vec{0})$ to avoid the generation of *chromomagnetic flux* hence,

$$\nabla \cdot [G(\phi) \mathbf{E}] = \bar{\psi} q \gamma^0 \psi = j^0 = \rho. \quad (5.5)$$

The electric field solution has the same expression as Eq.(4.6) with $q = (4/3)\pi R^3 \rho$. In spherical coordinates, Eq.(5.2) becomes

$$- \frac{1}{r^2} \frac{d}{dr} \left[r^2 \frac{d\phi}{dr} \right] - \frac{1}{2} \frac{\partial G(\phi)}{\partial \phi} E^2 + \frac{\partial V(\phi)}{\partial \phi} - \bar{\psi} m_q \psi \frac{\partial G(\phi)}{\partial \phi} = 0. \quad (5.6)$$

Using the electric field solution in Eqs.(4.6) and (4.7) into Eq.(5.6) yields,

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \frac{\partial}{\partial \phi} \left[V(\phi) + \frac{\lambda^2}{2} \frac{1}{V(\phi)} \frac{1}{r^4} - \bar{\psi} m_q \psi V(\phi) \right]. \quad (5.7)$$

Recollecting that $V(\phi) = G(\phi)$ and ignoring the term in the order of λ^2 , we can reduce the above expression to

$$\nabla^2 \phi = \frac{\partial V(\phi)}{\partial \phi} [1 - q m_q \delta(\vec{r})]. \quad (5.8)$$

Considering a static color charge distribution such that, $\psi(r) = \epsilon f(r)$ and $\bar{\psi}(r) = \bar{\epsilon} f(r)$, where ϵ and $\bar{\epsilon}$ are constant spinors and $f(r)$ is a Gaussian function, we can express $\bar{\psi} \gamma^0 \psi = \bar{\epsilon} \gamma^0 \epsilon f(r)^2 = \text{const} \cdot f(r)^2 \simeq \rho(r)$, therefore, $\bar{\psi} \psi = \bar{\epsilon} \epsilon f(r)^2 = \text{const} \cdot f(r)^2 = \rho(r)$. Hence, we can define $\bar{\psi} \psi \simeq q \delta(r)$ as substituted above.

Now, switching on the perturbation about the vacuum of the potential ϕ_0 for small fluctuations η by introducing a shift, $\phi \rightarrow \eta + \phi_0$, as employed in Sec. 4.1, Eq.(5.8) reads

$$\begin{aligned}\nabla^2(\phi_0 + \eta) &= \frac{\partial V(\phi)}{\partial \phi}(1 - qm_q\delta(\vec{r})) \\ \nabla^2\phi_0 + \nabla^2\eta &= \left(\frac{\partial V}{\partial \phi}|_{\phi_0} + \frac{\partial^2 V}{\partial \phi^2}|_{\phi_0}\eta \right) (1 - qm_q\delta(\vec{r})) \\ \Rightarrow \nabla^2\eta &= m_\phi^2(1 - qm_q\delta(\vec{r}))\eta;\end{aligned}\tag{5.9}$$

in the last step we have used the classical tachyon potential defined in Eq.(4.8) expanded similar to Eq.(4.9) to remove the tachyonic modes giving rise to the glueball potential.

5.2 Confining Potentials

To be able to calculate the confining potentials, we need to clearly define the nature of the Dirac delta function that appear in Eq.(5.9). Consequently, in the limit of step function [380], we will adopt

$$\delta(\vec{r}) = \begin{cases} \frac{1}{4\pi} \lim_{R \rightarrow 0} \left(\frac{3}{R^3} \right), & \text{if } r \leq R \\ 0, & r > R \end{cases};\tag{5.10}$$

here, R is the radius of the hadron and r is the inter-particle separation distance. Solving Eq.(5.9) for particles trapped inside the hadron i.e. $r \leq R$, we can express

$$\nabla^2\eta = (m_\phi^2 - qm_q^2)\eta;\tag{5.11}$$

where

$$R = \left(\frac{3m_\phi^2(2\pi\alpha')^2}{4m_q\pi} \right)^{1/3},\tag{5.12}$$

equivalent relations are reached in [49, 111] where the tachyon mass M^2 is inversely proportional to the Regge slope $M^2 \propto 1/\alpha'$ and $R \propto \sqrt{\alpha'}$. We introduce the Regge slope $(2\pi\alpha')$ for the sake of dimensional correction and clarity in representation. Thus, we can develop Eq.(5.11) as

$$\eta''(r) + \frac{2}{r}\eta'(r) + 2K\eta(r) = 0;\tag{5.13}$$

where $K = qm_q^2 - m_\phi^2$. This equation has several solutions, however, we will use two solutions corresponding to two different regimes i.e. IR and the UV regimes

$$\eta(r) = \frac{a \cosh(\sqrt{2|K|r})}{\alpha r \sqrt{|K|}} \quad \text{and} \quad \eta(r) = \frac{a \sin(\sqrt{2K}r)}{\alpha r \sqrt{K}} \quad \text{for} \quad r \leq R,\tag{5.14}$$

respectively. The solution at the left side of Eq.(5.14) will give the potential in the low energy regime only while the other in the right side will give rise to the potential in the UV regime with some contribution from the IR regime. These solutions will be treated separately in order to study the behaviour of the particles in each regime. Supposing a quark is kicked out of the hadron i.e. $r > R$, corresponding to $\delta(r) = 0$, Eq.(5.11) takes the form

$$\eta''(r) + \frac{2}{r}\eta'(r) + 2K_0\eta(r) = 0; \quad (5.15)$$

where $K_0 = -(1/2)m_\phi^2$ with solutions

$$\eta(r) = \frac{a \cosh(\sqrt{2|K_0|r})}{\alpha r \sqrt{|K_0|}} \quad \text{and} \quad \eta(r) = \frac{a \sin(\sqrt{2K_0r})}{\alpha r \sqrt{K_0}}, \quad \text{for } r > R. \quad (5.16)$$

A quark kicked out of the hadron in this manner behaves like a massless particle but remains confined to the hadron, this will be elaborated later in the subsequent sections.

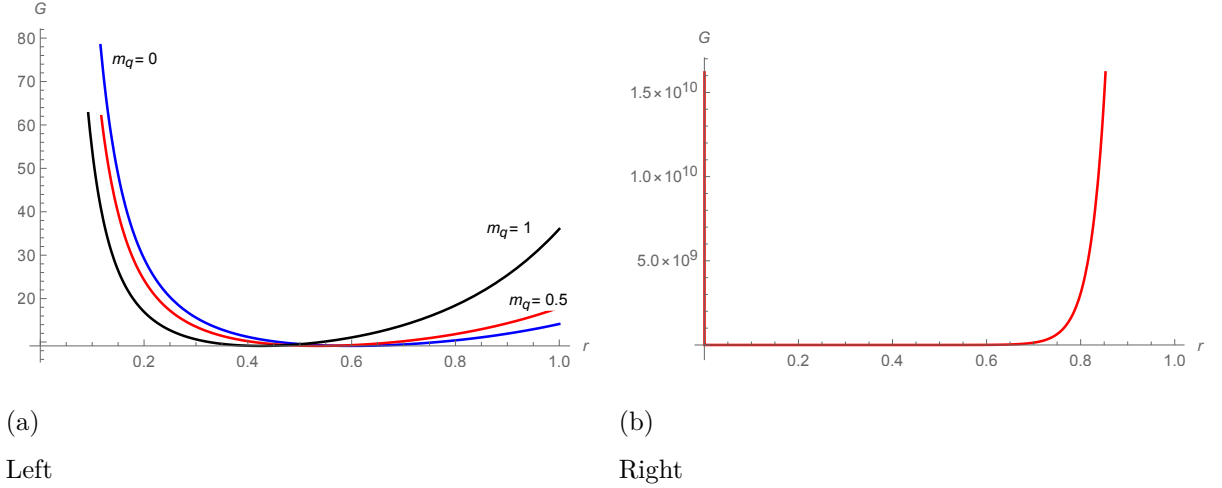
5.2.1 Net Confining Potential Inside the Hadron

The associated color dielectric function corresponding to the glueball potential can be obtained using similar approach as in Eq.(4.9) consequently,

$$\begin{aligned} G(\eta) &= \frac{m_\phi^2}{2}\eta^2 \\ &= \frac{\cosh^2(\sqrt{2|K|r})}{|K|r^2}; \quad \text{where } a^4\rho = 1. \end{aligned} \quad (5.17)$$

The particle-like state observed at $V''(\phi)|_{\phi_0}$ stabilizes the resulting QCD-like vacuum independent of the 'bare' quark mass through glueball condensation. For instance $|K| = m_\phi^2 - qm_q^2$, obviously when we put $m_q = 0$ the glueball potential still remain stable. This makes this type of potential suitable for studying both light and heavy quark systems.

Figure 5.1: A graph of glueball potential $G(r, m_q)$ against (r, m_q) for specific values of m_q (left) and infinite m_q (right).



The graph in the left panel shows an increase in glueball condensation with increase in depth as quark mass increases from $m_q = 0$ (blue), $m_q = 0.5$ (red) to $m_q = 1 = \Lambda_\chi$ (black). Likewise, in the right panel, the glueball condensation increases as $m_q \rightarrow \infty$.

Using the electric field solution (4.6) and the definition of the electrodynamic potential (4.14) with the solution in the IR regime, left side of (5.14) yields,

$$V_c(r, m_q) = \mp \lambda \frac{\sqrt{(m_\phi^2 - qm_q^2)} \tanh \left[\sqrt{2(m_\phi^2 - qm_q^2)} r \right]}{\sqrt{2}} + c. \quad (5.18)$$

We situate the analysis such that, the positive part of the potential correspond to a particle with a color charge $q = 1$ and the negative part of the potential will be classified as the potential for an antiparticle with an anticolor charge $q = -1$. So, two massive quark and an anti-quark pairs inside the hadron will always remain confined, because they will be attractive towards each other. Accordingly, (anti)quarks of the same flavour or polarity will be repulsive towards each other whilst (anti)quarks of different flavours or polarity will be attractive towards each other [1, 66]. For precision, we will select the potential of an antiparticle corresponding to $q = -1$ for detailed analysis. The relationship between q and the color charge g will be established explicitly in the subsequent sections. Thus, the net confining potential can be expressed as

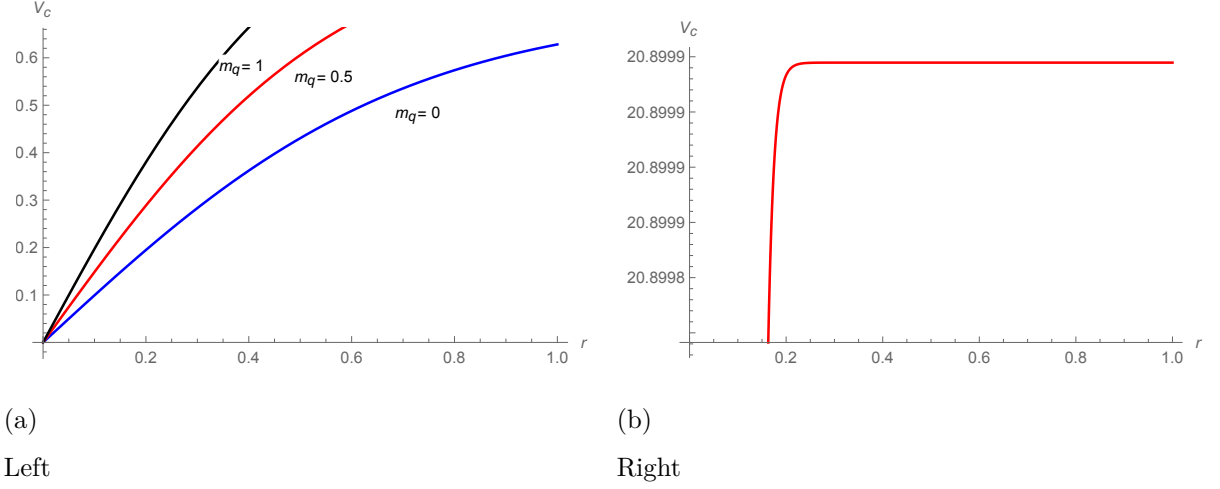
$$V_c(r, m_q) = \frac{\sqrt{(m_\phi^2 + m_q^2)} \tanh \left[\sqrt{2(m_\phi^2 + m_q^2)} r \right]}{\sqrt{2}}; \quad (5.19)$$

with string tension

$$\sigma_L(m_q) \simeq (m_\phi^2 + m_q^2) = m_L^2, \quad (5.20)$$

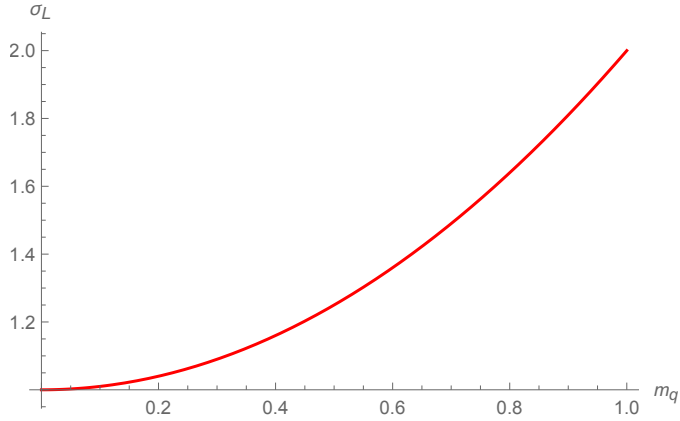
where m_L^2 is a glueball-meson mixing state [381, 382, 383, 384].

Figure 5.2: A graph of net confining potential, $V_c(r, m_q)$, against (r, m_q) for specific values of m_q (left) and an infinite m_q (right).



The graph shows the strength of confinement for different values of m_q , from $m_q = 0$ (blue), $m_q = 0.5$ (red) to $m_q = 1 = \Lambda_\chi$ (black) as their separation distance r varies. The increase in the gradient as m_q increases from 0 to $\Lambda_\chi = 1\text{GeV}$ shows stronger confinement with increasing m_q . The graph in the right panel is for infinite mass limit, it rises sharply, indicating confinement and flatten up shortly indicating hadronization phase.

Figure 5.3: The behaviour of the string tension σ_L against m_q .



The graph rises linearly with mass showing stronger confinement with increasing mass.

Since we used an antiparticle potential, $\lambda = -1$, since λ depends on q . The critical distance within which hadronization is observed is $r_* = 1/\sqrt{2\sigma}$ [342]. Moreover, the potential has both scalar contribution from m_q and a vector/gluon contribution at $m_q = 0$. In this low energy (IR) regimes colorless (baryons) or color neutral (mesons) particles

are expected due to *color confinement*. From the analysis in the previous chapters, we know that $\sigma \sim 1 \text{ GeV/fm}$, sticking to similar argument and bearing in mind that m_q can be small or zero but not negative. For instance, we can set $m_\phi^2 = 0.99 \text{ GeV}^2$ leading to $m_\phi = 995 \text{ MeV}$, which agrees with the scalar glueball mass for isoscalar resonance $f_0(980)$ reported in PDG to be $(990 \pm 20) \text{ MeV}$ [350]. This leads to a quark mass $m_q = 100 \text{ MeV}$, which falls within the regime of strange quark mass. Mass of a strange quark is projected to be $\bar{m}_s = (92.03 \pm 0.88) \text{ MeV}$ in the PDG report but higher results up to 400 MeV have been reached using other fit approaches [385]. Additionally, $\bar{m}_s(1 \text{ GeV}) = (159.50 \pm 8.8) \text{ MeV}$ was also reported in [158, 386]. As a result, this model will be efficient for investigating particles of masses within $0 \leq m \leq 100 \text{ MeV}$ hence, it will be suitable for investigating light quarks such as u, d and s. However, phenomenologically, quarks with masses less than $\Lambda_\chi = 1 \text{ GeV}$ are considered light and their characteristics can be studied under this model framework. In sum, this model is suitable for studying light quarks.

The potential can be interpreted as light spinless (scalar) quarks in slow motion around a heavy antiquark with mass m_q in the origin. So the light quarks that move close to the massive source gives rise to the relativistic spin-dependant correction whilst the ones revolving far from the source leads to relativistic spin-independent correction [157]. This combination gives rise to the net potential. Also, the heavy source m_q transform as color triplet. When the separation distance between the quarks exceed the threshold separation distance, there will be hadronization but none of the particles can be asymptotically free in this regime, so they recombine to form light mesons. It is worthy of noting that the quarks will remain 'imprisoned' since there exist at least a single heavy source inside the hadron. This is a common phenomenon in HQET [1].

On the other hand, we will investigate the UV potential using the solution at the right side of (5.14), giving rise to

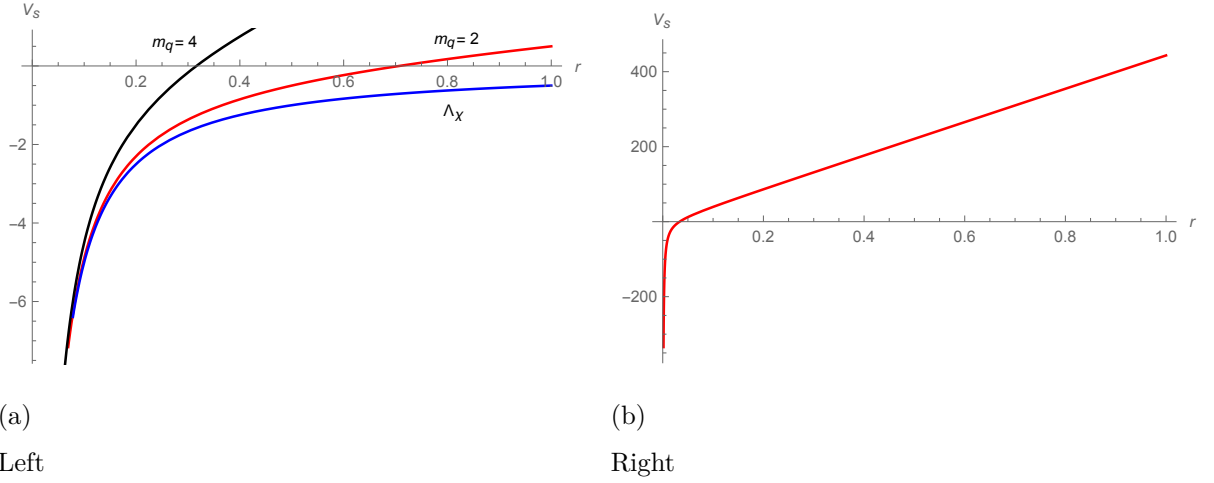
$$\begin{aligned}
V_s(r, m_q) &= -\frac{\sqrt{K}\lambda \cot(\sqrt{2K})r}{\sqrt{2}} + c \\
&\simeq -\frac{1}{2r} + \frac{K}{3}r + c \\
&\simeq \left[-\frac{1}{2r} + \frac{(m_q^2 - m_\phi^2)r}{3} + \mathcal{O}(r^3) \right] + c,
\end{aligned} \tag{5.21}$$

with string tension

$$\begin{aligned}\sigma_s &= \frac{m_q^2 - m_\phi^2}{3} \\ &= \frac{m_s^2}{3}.\end{aligned}\tag{5.22}$$

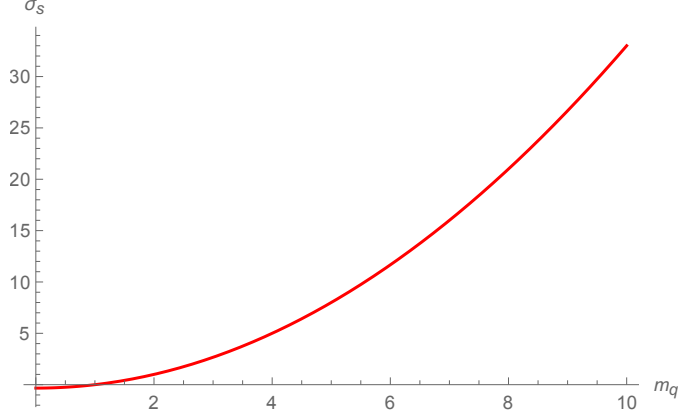
Here, we chose the positive part of the potential corresponding to particles with $\lambda = q = 1$, and m_s^2 is the glueball-meson mixing state in this regime. This potential shows both the asymptotic free nature of QCD theory represented by the Coulomb-like part of Eq.(5.21) and a *color confining* part also depicted by the linearly rising part of the potential.

Figure 5.4: A graph of the Cornell-like potential $V_s(r, m_q)$ against r, m_q for specific values of m_q (left) and infinite m_q limit (right).



The graph in the left panel rises steadily with increasing mass from $m_q = 1$ (blue), $m_q = 2$ (red) to $m_q = 4$ (black). Indicating strong confinement with increasing mass whilst the graph in the right panel shows stronger confinement with infinite mass.

Figure 5.5: A graph of string tension $\sigma_s(m_q)$ against quark mass m_q .



The graph shows strong confinement with increasing quark mass m_q , however, the string is unstable at $m_q = 0$ and degenerate at $m_q = m_\phi$, thus confinement begins at $m_q > 1$.

Furthermore, knowing the magnitude of the string tension and m_ϕ^2 , we can deduce that, $m_q \simeq 2 \text{ GeV}$, and the bound state of particle and antiparticle state becomes $m_q^2 \simeq 4 \text{ GeV}^2$. Such particles can be comparable with the mass of a heavy quark such as the charm quark $\bar{m}_c(\bar{m}_c) = (1.280 \pm 0.025) \text{ GeV}$ reported in the PDG, and higher values up to $m_c = 1.49 \text{ GeV}$ discussed in [385]. In the model framework, $m_q > m_\phi$, in order to obtain confinement in the UV regime. Therefore, charmonium properties can be investigated under this model. Besides, at some critical mass m there will be hadronization. Thus, the model works well under the restriction $m_\phi < m_q \leq m$. Finally, light quarks are difficult to determine due to their masses compared to the hadron size, therefore, their classification and characteristics are difficult to observe in the framework of a hadron model.

5.2.2 String Tension

Cornell potential can be expressed in broader terms as

$$V(r) = -\frac{e}{r} + \sigma r + V_0; \quad (5.23)$$

where e and σ are positive constants whose magnitude can be determined through fit. However, σ is related to the lattice spacing a , through $\sigma \sim 1/a^2$, and V_0 is the self-energy of the static quark. Wider lattice spacing implies large separation distance r [56]. The magnitude of e can only be determined phenomenologically through fit. Besides,

its magnitude has an inverse relationship with the mass of the particle under investigation. The model studied here involves a mass of $m_q = 2 \text{ GeV}$ and $e = 0.5$ compared to the results obtained in [1] where e was determined to be 0.25 [342] corresponding to a mass of $m_q = 4 \text{ GeV}$. In the model discussed in this chapter, baryon number is conserved due to the presence of 'bare' quark mass and a constituent quark mass function. However, in quenched models, the conserved quantity is the individual particle and antiparticles present. The QCD-like vacuum associated with this model involves haphazard creation and annihilation of particles. This is associated with an excitation in the string tension connecting the particles together leading to transitions in the string tension and hadronization. When the separation distance between static quark and an antiquark pairs exceeds some separation distance r_{*s} or the quark mass exceeds some critical mass m , the string tension will degenerate leading to formation of more light mesons. This affects the value of e since light quarks implies greater e [158]. This leads to the screening of m_q by the light mesons created in the process. We have also found that $\sigma_L > \sigma_s$ at the ground state where there are no quarks. So, light quarks are more likely to be confined with σ_L than σ_s . As a result, the quark mass associated with σ_L lies within $0 \leq m_q \leq \Lambda_\chi$ whilst σ_s is within $2 \leq m_q \leq m \text{ GeV}$. Since the results obtained here are compared with a mass of a hadron 1 GeV with radius 1 fm . We can compare the results with the ones obtained in [386] where $\bar{m}_u(1 \text{ GeV}) = (5.1 \pm 0.9) \text{ MeV}$, $\bar{m}_d(1 \text{ GeV}) = (9.0 \pm 1.6) \text{ MeV}$, $\bar{m}_s(1 \text{ GeV}) = (161 \pm 28) \text{ MeV}$ [386, 387, 158] and other related values contained in [388]. Meanwhile, heavier counterparts can be compared with [158] where $\bar{m}_c(1 \text{ GeV}) = (1.40 \pm 0.06) \text{ GeV}$ and $\bar{m}_b(1 \text{ GeV}) = (5.87 \pm 0.06) \text{ GeV}$.

5.2.3 Vector Potential

The quark kicked out of the hadron have only *vector potential*, because at that instant, $m_q = 0$, so its associated potential have only gluon contributions. Since the quark remain confined to the hadron, its net potential will have contribution from the hadron i.e. *scalar potential*. But we will deal with the *vector potential* here, whilst we leave the *scalar potential* to be dealt with latter. It is important to notice that since the hadron serves as a massive point-like source, the 'break away' quark will always remain in 'prison' to the hadron. In that light, it has an unstable UV contribution which can be seen from the solution at the right side of Eq.(5.16). Consequently, the vector confining

potential calculated from the solution on the left side of Eq.(5.16) yields,

$$\begin{aligned} V_v(r) &= \mp \frac{\lambda \sqrt{|K_0|} \tanh[\sqrt{2|K_0|r}]}{\sqrt{2}} + c \\ &= \mp \frac{\lambda m_\phi \tanh(m_\phi r)}{2} + c, \end{aligned} \quad (5.24)$$

with string tension

$$\sigma_V = \frac{m_\phi^2}{2}. \quad (5.25)$$

This result can also be interpreted as a consequence of short distance relativistic spin correction.

5.2.4 Scalar Potential

As alluded to in the preceding subsection, in order to properly study how the 'break away' quark remain confined to the hadron, we need to know the scalar contribution from the hadron. This will be the main objective of this subsection. Under the circumstance, we will juxtapose the result in Eq.(5.3) with the generalized Dirac's equation where both the vector and the scalar potentials coexist i.e.,

$$[c\hat{\alpha}\hat{p} + \hat{\beta}m_0c^2 + V(r)]\psi = 0. \quad (5.26)$$

Where the four momentum and the gauge fields can be decomposed as $p^\mu = (E/c, \vec{p})$ and $A^\mu = (\Phi(r), \vec{A})$. Since the hadron under study is considered static and scalar (spinless) in form, we can set $E = \vec{A} = \Phi(r) = 0$, these considerations were made in writing Eq.(5.26). For simplicity without ignoring generality, we can restrict the potential to a spherical wall such that

$$V(r) = \begin{cases} S(r), & \text{for } r \leq R \\ 0, & \text{for } r > R \end{cases}; \quad (5.27)$$

where r is the inter-particle separation distance and R is the radius of the hadron. So, we can write the Dirac's equation as

$$(c\hat{\alpha}\hat{p} + \hat{\beta}m_0c^2)\psi + S(r)\psi = 0; \quad (5.28)$$

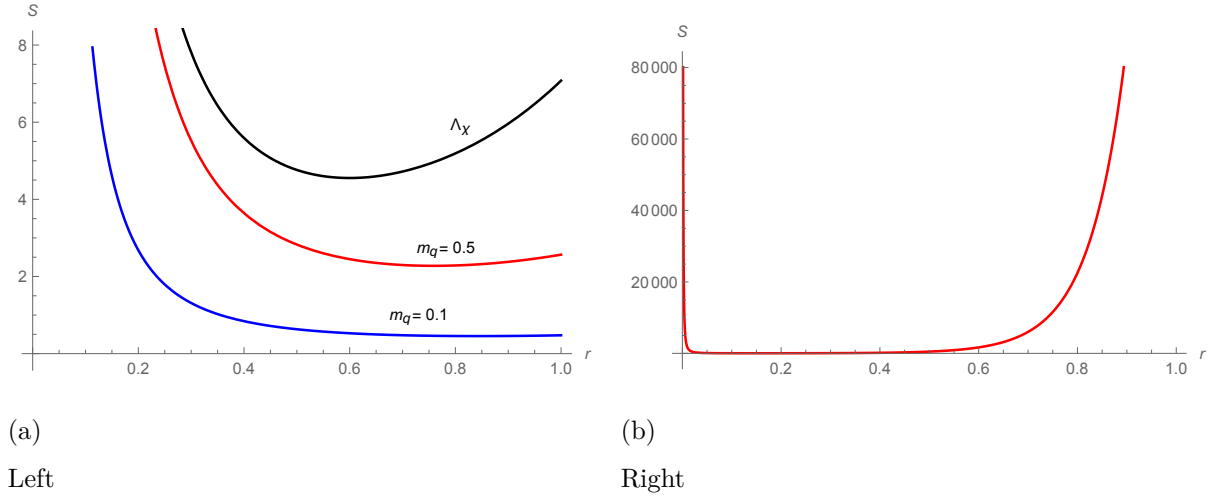
where $\hat{\alpha}$ and $\hat{\beta}$ constitute Dirac matrices, $\hat{p} = -i\nabla$ is the momentum operator and m_0 is the rest mass of the hadron. The vector potential of the particles can be reintroduced

by minimum substitution of the four momentum $P_\mu \rightarrow P_\mu - gA_\mu$ and the scalar potential was introduced as $m \rightarrow m_0 + S$, where g is a positive coupling constant. That notwithstanding, the vector and the scalar potentials have the same weight but coupled differently in the Dirac equation [396]. Comparing Eqs.(5.3) and (5.28) we can identify the scalar potential $S(r)$, while the remaining terms in both equations are the interaction terms [397, 398]. Accordingly, the scalar potential can be expressed from Eq.(5.17) as

$$\begin{aligned} S(r) = m_q G(r) &= \frac{m_q \cosh^2 \left(\sqrt{2|K|r} \right)}{|K|r^2} \\ &= \frac{m_q \cosh^2 \left[\sqrt{2(m_\phi^2 + m_q^2)r} \right]}{(m_\phi^2 + m_q^2)r^2}, \end{aligned} \quad (5.29)$$

we set $q = -1$ in this computations. This potential is akin to long range relativistic spin corrections. Meaning, the short range relativistic spin-orbit interactions are overshadowed giving way to scalar interactions only. In this case, the interactions are seen to be centred on the hadron or the quark coordinates. This is the scalar potential energy of the hadron. Besides, its relationship with the glueball potential $G(r)$ indicates that high glueball condensation implies stronger confinement and the vice versa. Additionally, at $m_q = 0$ the potential simply vanishes therefore, this type of potential cannot be used to confine particles in a system with no fermions.

Figure 5.6: Variation of scalar potential $S(r, m_q)$ with (r, m_q) for specific values of m_q (left) and infinite m_q (right).



The potential is near flat at $m_q = 0.1$, and increases in magnitude as m_q increases, from $m_q = 0.1$ (blue), $m_q = 0.5$ (red) to $m_q = 1 = \Lambda_\chi$ (black). The curve decreases with distance until it attain its minimum at $S(r, m_q) \rightarrow 0$ and start rising from the minimum towards the right. The decrease implies confinement while the rise towards the right signifies degeneracy and screening of the static antiquark source [342].

5.2.5 Net Confining Potential Energy

Indeed, some models have predicted the possibility of *vector potential* dominance over the *scalar potentials* in systems where both potentials coexist. Because, not all quark combinations are expected to be confined in such systems. Therefore, the most probable combinations are the energetic ones as determined phenomenologically [66, 1]. We will contribute to this study by calculating the couplings of the vector and the scalar potentials and compare their differences to determine the strength of the potentials. The *net potential energy* of the 'break away' quark can be expressed as the sum of Eqs.(5.24) and (5.29) accordingly,

$$V_{\text{net}}(r) = \mp \frac{q^2 m_\phi \tanh(m_\phi r)}{8\pi} + \frac{m_q \cosh^2 \left[\sqrt{2(m_\phi^2 + m_q^2)} r \right]}{(m_\phi^2 + m_q^2) r^2}. \quad (5.30)$$

Here, we have substituted $\lambda = -q/4\pi$ and multiply the vector potential by $-q$ in order to obtain the expected *net potential energy*.

5.2.6 Constituent Quark Masses

Constituent quark masses are commonly associated with phenomenological models used in investigating nonperturbative aspects of QCD theory. We identified constituent quark mass functions $M(\phi) = m_q G(\phi)$ below Eq.(5.1). In this section we will throw more light on it and seek to estimate its magnitude at the ground state $r \rightarrow r_*$ and at the higher excited state $r \rightarrow 0$. Again, the constituent quark mass function intended for discussion in this subsection is similar to the scalar potential for arbitrary separation distance r discussed in Sec. 5.2.4. We will determine the ground state constituent mass from the IR solution in Eq.(5.14) and the mass for the highest excited state determined from the UV solution. Using the solution at the left side of Eq.(5.14) and expanding it for $\sqrt{2|K|r} \ll 1$ we obtain

$$\begin{aligned}\eta(r) &= \frac{\cosh(\sqrt{2|K|r})}{r\alpha\sqrt{2|K|}} \\ &\simeq \frac{1}{r\alpha\sqrt{2|K|}} \left[1 + |K|r^2\right] + \mathcal{O}(r^3).\end{aligned}\tag{5.31}$$

Introducing the definition of $G(\eta)$ Eq.(5.17) into the equation and multiplying it by m_q to obtain $M(r)$ gives

$$\begin{aligned}M(r) &= \frac{m_q}{r^2|K|} \left[1 + 2|K|r^2\right] + \mathcal{O}(r^3) \\ &= \frac{m_q}{r^2|K|} + 2m_q + \mathcal{O}(r^3).\end{aligned}\tag{5.32}$$

Thus, at the ground state $r \rightarrow r_*$ the constituent quark mass will be $M_* = 2m_q$ [342], as discussed earlier, r_* is the distance within which the quarks are confined, beyond this threshold, they begin to hadronize. Accordingly, the constituent quark mass suitable for this regime will be $M_* = 200 \text{ MeV}$.

We now analyse the constituent quark mass using the solution at the right side of Eq.(5.14) following similar procedure yields,

$$M(r) = 2m_q \left[1 - \frac{2Kr^2}{3}\right] + \mathcal{O}(r^3),\tag{5.33}$$

in the limit $r \rightarrow 0$ i.e. the highest excited state, we get $M_0 = 2m_q$. Hence, the constituent mass in this regime will be $M_0 = 4 \text{ GeV}$. In the model framework, the constituent mass (M) falls within $0 \leq M \leq 4 \text{ GeV}$. The IR behaviour can be investigated within $0 \leq M_* \leq 200 \text{ MeV}$, this falls within the range of light quarks, phenomenologically.

Whilst the UV regime takes on masses within the range of $2 \leq M_0 \leq 4 \text{ GeV}$, suitable for investigating *charmonium* and *bottomonium* properties. Particles that fall inbetween 200 MeV and 1 GeV can preferably be studied under the IR potential whilst particles of masses greater than 1 GeV can be studied under the UV potential.

By the same token, the *constituent quark masses* [336] are known to break down the chiral symmetry *dynamically* or *explicitly*, hence it is important to distinguish the regimes for which these phenomenon occur separately. Obviously, these are nonperturbative consequences of QCD theory. Subsequently, we will set the threshold for *dynamical chiral symmetry breaking* at Λ_χ , corresponding to a maximum value of 1 GeV [79]. By convention, the quarks are described as heavy when their masses are $M > \Lambda_\chi$, this regime is classified under *explicit chiral symmetry breaking*. Therefore, the model under study will lead to *dynamical chiral symmetry breaking* in the IR regime and *explicit chiral symmetry breaking* in the UV regime respectively. As a result, *spontaneous chiral symmetry* is observed at $M < \Lambda_\chi$ with the presence of light flavour quarks. In nonrelativistic QCD models where constituent quark models are commonly used, projects the constituent quark masses for up and down quarks as $M_u = M_d = 350 \text{ MeV}$ in single gluon exchange interactions, and a charm quark as $M_c = 1.6 \text{ GeV}$. Their magnitudes are greater than the usual *current quark masses*, and they vary depending on how measurements are made in the model framework. They are phenomenologically free parameters that can be determined through fits. Some of the fits used in determining such masses are; meson fits, baryon fits, hadron fits or other phenomenological potential models.

5.2.7 Glueball Masses

We will present the calculations for glueball mass and glueball-meson mixing states and analyse them accordingly. The lightest scalar glueball mass is known to be within 1.5 to 1.7 GeV [28], besides, a specific value of 1.7 GeV was reached using unquenched calculations [399], i.e. QCD with dynamical quarks. On the other hand, vector glueball mass also exist with estimated value of 3.8 GeV through quenched lattice QCD calculations i.e. QCD without dynamic quarks [29]. Some facilities such as Beijing Spectrometer Experiment (BESIII) and PANDA experiment at the FAIR Lab [397] have been on a search for this particle. However, BESIII facility had a positive outcome with an observed vector particle of magnitude 3.77 to 4.60 GeV with precision [400]. We calculate the

glueball mass from the fluctuations in the tachyon potential

$$m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2} \Big|_{\phi_0} = 2a^2 \alpha^2 \rho. \quad (5.34)$$

This has been fixed within the model framework to be $m_\phi = 995 \text{ MeV}$. Also, we can obtain unquenched glueball mass (glueball-meson mixing state) [381, 382, 383, 384] from the Lagrangian density (5.1) i.e.,

$$m_{\phi H}^2 = - \left\langle \frac{\partial^2 \mathcal{L}}{\partial \phi^2} \right\rangle \Big|_{\phi_0} = m_\phi^2 (1 - q\delta(r)m_q) + \frac{m_\phi^2}{4} \langle F^{\mu\nu} F_{\mu\nu} \rangle \Big|_{\phi_0}, \quad (5.35)$$

in the classical picture $\langle F^{\mu\nu} F_{\mu\nu} \rangle \Big|_{\phi_0} = 0$ [1] as discussed in Sec. 4.4 consequently,

$$m_{\phi H}^2 = m_\phi^2 - qm_q^2. \quad (5.36)$$

When we choose $q = -1$, we get $m_L^2 = m_\phi^2 + m_q^2$, Eq.(5.20), and we retrieve $m_s^2 = m_q^2 - m_\phi^2$, Eq.(5.22), when we substitute $m_\phi^2 \rightarrow -m_\phi^2$, the glueball-meson mixing state in the UV regime. This is in line with the behaviour of nonperturbative QGP phase where the glueballs and the gluon condensate change sign in the UV regime. This is evident in lattice $SU(3)_c$ gauge calculations [432, 433]. The magnitude for m_L and m_s were also determined from the model framework to be 1 GeV and 1.73 GeV respectively.

5.2.8 Coupling Constants

This section will be useful to this chapter, Cap. 4 and the subsequent chapters as well, because we will reveal the relationship between q and color charge g . This will be achieved by comparing the result obtained in Eq.(5.7) with the expressions reached in [66] using dilaton in gauge theory to confine quarks and gluons. Similar approach was adopted in [1, 2, 317]. This will enable us determine the coupling of the vector and the scalar potentials, so their strengths can be analysed. The expression in [66] is given by

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = - \frac{g^2}{64\pi^2 f_\phi} \left(1 - \frac{1}{N_c}\right) \exp\left(-\frac{\phi}{f_\phi}\right) \frac{1}{r^4} - \frac{\xi}{2f_\phi} \exp\left(-\xi \frac{\phi}{2f_\phi}\right) m_q g \delta(\mathbf{r}). \quad (5.37)$$

The potential used in this equation is exponential, so to make it suitable for comparison with the model discussed in this thesis, we will transform the potential to conform with the tachyon potential adopted for this work. Therefore,

$$\exp\left(-\frac{\phi(r)}{f_\phi}\right) \rightarrow 2(\alpha^4 \phi^3 - \alpha^2 \phi) \quad \text{and} \quad \exp\left(-\xi \frac{\phi(r)}{2f_\phi}\right) \rightarrow 2\left(\frac{\alpha^4 \phi^3 \xi^4}{16} - \frac{\alpha^2 \phi \xi^2}{4}\right), \quad (5.38)$$

as a result, Eq.(5.37) becomes

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \frac{g^2}{32\pi^2 f_\phi} \left(1 - \frac{1}{N_c}\right) \frac{\alpha^2\phi}{r^4} + \frac{2\xi^3\alpha^2\phi}{8f_\phi} m_q g \delta(\mathbf{r}). \quad (5.39)$$

In the same vein, we can rewrite Eq.(5.7) as

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = -2\alpha^2\phi + \frac{\alpha^2\lambda^2\phi}{r^4} + 2qm_q\alpha^2\delta(r)\phi. \quad (5.40)$$

In writing these expressions we choose only the linear terms in ϕ in order to make it easy for comparison. To proceed with the comparison, we set the mass decay constant $f_\phi = 1/\alpha^2$, where $f_\alpha = 1/\alpha$ is the tachyon decay constant. Now juxtaposing Eq.(5.39) with Eq.(5.40), we can identify

$$\lambda^2 = \frac{g^2}{32\pi^2} \left(1 - \frac{1}{N_c}\right) \rightarrow q = g\sqrt{\left(1 - \frac{1}{N_c}\right)}, \quad (5.41)$$

here, we substituted λ with Eq.(4.7) and $g \rightarrow g/\sqrt{2}$. Also,

$$\frac{\xi^3}{8} = 1 \rightarrow \xi = 2, \quad (5.42)$$

which is the fermion coupling to the equation. In the 't Hooft limit $N_c \rightarrow \infty$, $g = q$, in most cases this produces similar results as $N_c = 3$ i.e. particles with 3 flavours, this runs through the whole thesis.

From the afore analyses, we observe that the net potential in Eq.(5.19) constitutes no mass coupling $\xi = 0$ and *Kaluza-Klein-type* or mass coupling, $\xi = 1$ [401]. The mixture of this two couplings gives rise to the net confining potential of any particle confined inside the hadron. While the coupling constant for the *scalar potential energy* Eq.(5.29) is $\xi = 2$ and the coupling for the *vector potential* Eq.(5.24) is $\xi = 0$. The higher coupling of the *scalar potential energy* shows its weakness relative to the *vector potential*. And so, it must be coupled strongly in order to coexist with the *vector potential* which needs no coupling at all. Thus, the *scalar potential* is weaker than the *vector potential* by a ratio of 0 : 2. In terms of the strength of the potentials, the *scalar potential* vanishes at $m_q = 0$ while the *vector potential* is insensitive to the quark mass. Accordingly, the quarks remain confined [14, 402, 403] in net confining potential Eq.(5.19) and the net confining *potential energy* Eq.(5.30) for any arbitrary quark mass m_q . However, at $m_q = 0$ the scalar parts of the potentials vanishes leaving only the vector part, so both equations become equal due to the *chromoelectric flux* confinement.

5.3 Quark and Gluon Condensation

This section will be an extension of Sec. 4.4.2 with an introduction of fermion fields. Consequently, the trace of the energy momentum tensor of the Lagrangian density (5.1) using the relation (4.47) yields,

$$\begin{aligned}\theta_\mu^\mu &= 4V(\eta) - \eta \frac{\partial V}{\partial \eta} - \frac{\eta}{4} \frac{\partial G}{\partial \eta} F^{\mu\nu} F_{\mu\nu} + q\delta(\vec{r})m_q\eta \frac{\partial G}{\partial \eta} \\ &= 4\tilde{V}' + \tilde{G}' F^{\mu\nu} F_{\mu\nu} - 4q\delta(\vec{r})m_q\tilde{G}' \\ &= 4\tilde{V}'_{eff} + \tilde{G}' F_{\mu\nu} F^{\mu\nu}.\end{aligned}\tag{5.43}$$

Where we have introduced the expressions

$$\tilde{V}' = V - \frac{\eta}{4} \frac{\partial V}{\partial \eta}, \quad \tilde{G}' = -\frac{\eta}{4} \frac{\partial G}{\partial \eta} \quad \text{and} \quad \tilde{V}'_{eff} = \tilde{V}' - q\delta(\vec{r})m_q\tilde{G}',\tag{5.44}$$

to make the computations simple and more compact. Juxtaposing this with the expectation value of the trace of the energy momentum tensor expressed in Eq.(4.45), we get

$$\langle \tilde{G}'(\eta) F^{\mu\nu} F_{\mu\nu} \rangle = -4(|\epsilon_v| + \tilde{V}'_{eff}(\eta)).\tag{5.45}$$

Rescaling $\tilde{V}'_{eff}$ with the vacuum energy density $-|\epsilon_v|$ as

$$\tilde{V}'_{eff}(\eta) \rightarrow -|\epsilon_v|\tilde{V}'_{eff},\tag{5.46}$$

gives rise to the expression

$$\langle \tilde{G}'(\eta) F^{\mu\nu} F_{\mu\nu} \rangle = 4|\epsilon_v|\langle \tilde{V}'_{eff} - 1 \rangle.\tag{5.47}$$

This equation agrees with the vanishing of the condensate in the classical limit where $|\epsilon_v| \rightarrow 0$ or in terms of classical tachyon field ϕ , $\langle F^{\mu\nu} F_{\mu\nu} \rangle|_{\phi_0} = 0$. Because $\tilde{V}'_{eff}(\phi)|_{\phi_0} = 0$ and $\tilde{G}'(\phi)|_{\phi_0} = 0$, when calculated explicitly from the potential (4.8). The gluon condensation becomes non-vanishing when the glueball field η and its potential $V(\eta)$ are used. Since, quark and gluon condensations are an IR property of the QCD theory, we will choose a suitable gluon charge q to achieve the desired result. Bearing in mind that $V = G$, we can express

$$\begin{aligned}\tilde{V}'_{eff}(\eta) &= \tilde{V}' - q\delta(r)m_q\tilde{G}' \\ &= V - \frac{\eta}{4} \frac{\partial V}{\partial \eta} - qm_q\delta(r) \left(-\frac{\eta}{4} \frac{\partial G}{\partial \eta} \right) \\ &= V - \frac{\eta}{4} \frac{\partial V}{\partial \eta} (1 - qm_q\delta(r)).\end{aligned}\tag{5.48}$$

Introducing $G(\eta) = (1/2)m_\phi^2\eta^2$ yields,

$$\begin{aligned}\tilde{V}'_{eff}(r, m_q) &= \frac{m_\phi^2}{4}\eta^2 - q\delta(r)m_q \left(-\frac{m_\phi^2}{4}\eta^2\right) \\ &= \frac{1}{4} \left(m_\phi^2 + m_q^2\right) \eta^2 \quad \text{for } q = 1, \\ &= \frac{1}{4}m_L^2\eta^2 = \left(\frac{\cosh[\sqrt{2(m_\phi^2 + m_q^2)}r]}{\sqrt{2}m_\phi r}\right)^2.\end{aligned}\tag{5.49}$$

By this effective glueball potential we have more glueballs condensing at $\tilde{V}'_{eff}(r, m_q) \rightarrow 0$ i.e. $\eta \rightarrow 0$. Accordingly, the condensate expressed in Eq.(5.45) takes the form

$$\langle G(\eta)F^{\mu\nu}F_{\mu\nu} \rangle = 8|\epsilon_v| \left\langle 1 - \frac{m_L^2\eta^2}{4} \right\rangle;\tag{5.50}$$

where the maximum condensate is achieved at $\eta \rightarrow 0$. Additionally, the condensate persists even at $m_q = 0$ due to the presence of a stable glueball mass. The gluon mass associated with this condensate can be derived as

$$m_{AL}^2 = \frac{m_L^2}{4}.\tag{5.51}$$

Generally, the glueball mass is related to the gluon mass by

$$\frac{m(0^{++})}{m_A} = \sqrt{6} \simeq 2.45,\tag{5.52}$$

considering the leading term obtained from Yang Mills theory with an auxiliary field ϕ [404, 3]. In that case, the scalar glueball mass $m(0^{++})$ is derived from the fluctuations around ϕ . On the other hand, gluon mass obtained from QCD theory [405] with no dynamic quarks have also been estimated as

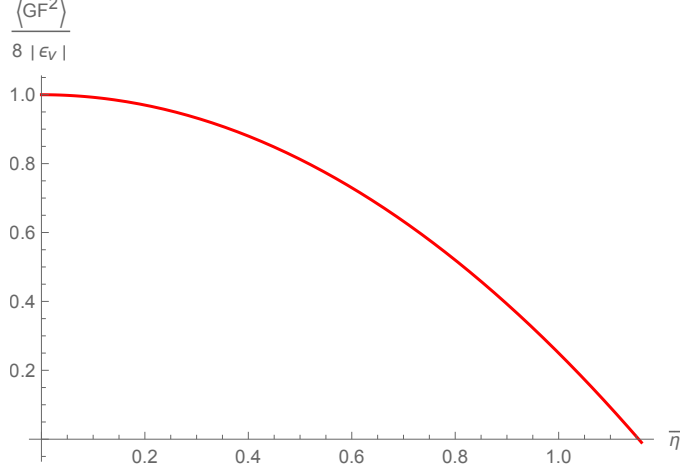
$$m_g^2 = \left(\frac{34N\pi^2}{9(N^2 - 1)} \left\langle \frac{\alpha_s}{\pi} G^{\mu\nu} G_{\mu\nu} \right\rangle\right)^{1/2};\tag{5.53}$$

here, N and α_s are the color number and the strong coupling constant respectively. Again, this equation relates the gluon mass to the gluon condensate [406, 407].

The gluon mass is occasioned due to screening of the color charges and the bound states formed through self interactions among gluons. Its value have been estimated through QCD lattice simulation to be in the region of $m_A = 600 \sim 700$ MeV [408]. Heavier ones in the range of ~ 1 GeV have been determined through phenomenological analyses [409] and other analytical methods [410]. Since the glueball-meson mixing state in the IR regime is estimated to be $m_L^2 \simeq 1 \text{ GeV}^2$, we can determine the gluon mass to

be $m_{AL} \simeq 500 \text{ MeV}$. Likewise, using the glueball-meson mixing state $m_s = 3 \text{ GeV}^2$ in the UV regime leads to $m_{AL} \simeq 870 \text{ MeV}$.

Figure 5.7: A graph of gluon condensate $\langle G(\eta)F^{\mu\nu}F_{\mu\nu} \rangle / (8|\epsilon_v|)$ against mean glueball field $\bar{\eta}$.



The condensation has its maximum value at $\bar{\eta} = 0$ and reduces steadily with increasing $\bar{\eta}$ until it vanishes at $\bar{\eta} \simeq 1$. Hence, the glueball field η has two possible solutions

5.3.1 Identifying the Strong Running Coupling α_s and the β -Function

Simplifying Eq.(5.43) leads to the expression

$$\theta_\mu^\mu = m_L^2 \eta^2 - \frac{\eta G'(\eta)}{4} F^{\mu\nu} F_{\mu\nu}. \quad (5.54)$$

Now, comparing the above expression with the trace of the QCD energy momentum tensor θ_μ^μ derived from the classical chiral effective QCD Lagrangian [411, 412],

$$\theta_\mu^\mu = \sum_f m_f \bar{q}_f q_f - \frac{b\alpha_s}{8\pi} F^{a\mu\nu} F_{\mu\nu}^a; \quad (5.55)$$

where $\beta(g) = -b\alpha_s/(4\pi)$, $b = 11$ (for pure gluodynamics), m_f is the *current quark mass* and q_f is the *quark field*. From the above equations we can identify $\sum_f m_f = m_L^2$, $\sum_f \bar{q}_f q_f = \eta^2$ and $b\alpha_s/2(4\pi) = \eta G'(\eta)/4 = -\beta(1/r^2)/2$. Employing the renormalization group theory [190]

$$\beta(Q^2) = Q^2 \frac{d\alpha_s(Q^2)}{dQ^2}, \quad (5.56)$$

and comparing it to

$$\beta(\eta) = -\eta \frac{d}{d\eta} \left(\frac{1}{2} G(\eta) \right) = -\frac{m_\phi^2}{2} \eta^2 = -\beta(1/r^2), \quad (5.57)$$

the strong running coupling can be determined as

$$\alpha_s(\eta) = \frac{1}{2} G(\eta) = \frac{m_\phi^2 \eta^2}{4} = \alpha_s(1/r^2). \quad (5.58)$$

It is worth noting that the QCD β -function is naturally a negative quantity.

5.4 Analysis

Vector potential possess' an angular momentum of the quarks whilst the *scalar potential energy* do not. The *scalar potential energy* is determined on the grounds that the angular momentum of the quarks are eliminated. In the *vector potential*, the interaction between the quarks are carried by the associated string tension. With the *scalar potential energy* on the other hand, the interaction is concentrated on the quark coordinates. The *scalar potential energy* is mostly applied in non-rotating quark systems whilst the *vector potential* is dominant in rotating quark systems. Accordingly, the relativistic spin corrections (Thomas Precession) that lead to vector confinement is quite distinct from the one that leads to scalar confinement [413, 414, 370, 371, 372].

We have calculated the confining potentials of quarks inside the hadron and when the quark is kicked out of the hadron. In the case of the particle kicked out of the hadron, the 'break away' quark behaves like a bubble and remains confined to the hadron. We also established that the glueball condensation leads to gluon condensation and color confinement. The glueball condensation increases with increasing quark mass m_q similar to what is seen with confinement of fermions in classical QCD theory. Additionally, the scalar potential energy is identified to be weaker than the vector potential in the model framework, because the scalar potential energy needs 100% coupling in order to coexist with the vector potential energy which needed no coupling at all. The ratio of the coupling ξ for the vector to the scalar potential energy is given as 0 : 2. The scalar potential vanishes at $m_q = 0$ whilst the vector and the net potentials persist due to gluon confinement.

Furthermore, the confining potentials in the IR and the UV regimes are associated with critical separation distances within which confinement is observed, i.e. $r_* =$

$1/\sqrt{2\sigma_L} \simeq 0.707 \text{ fm}$ and $r_{*s} = 1/\sqrt{\sigma_s} \simeq 1 \text{ fm}$ respectively. We also dealt with gluon condensation, glueball mass, glueball-meson mixing states and constituent quark masses. We observed that the gluon condensate is associated with gluon masses $m_{AL} \simeq 500 \text{ MeV}$ and $m_{AL} \simeq 870 \text{ MeV}$ in both the IR and UV regimes respectively. The observed glueball mass was determined to be $m_\phi \simeq 995 \text{ MeV}$ whilst the glueball-meson mixing states determined were $m_L = 1 \text{ GeV}$ and $m_s \simeq 1.73 \text{ GeV}$ corresponding to the ratios $m_L/\sqrt{\sigma_L} = 1$ and $m_s/\sqrt{\sigma_s} = 1.73$ in the IR and the UV regimes respectively. The observed constituent quark masses were $M_* = 2m_q = 200 \text{ MeV}$ and $M_0 = 2m_q = 4 \text{ GeV}$ corresponding to the IR and UV regimes respectively.

Chapter 6

Confinement of Fermions at Finite Temperature

High temperature study of heavy quarkonia is significant for both theoretical and experimental study of QCD matter and by extension QGP [415]. This study gained grounds after the production of heavy quarkonia such as charmonium bound states in RHIC [358] and LHC [47], giving strong evidence for investigating QGP phase transition. In the study of lattice QCD, it has been projected that J/ψ bound states may persist even if the critical temperature T_c of the system is exceeded, $T = 1.5 T_c$ [360]. Besides, in simple models such as the one intended for this chapter, $T = T_c$ represent deconfinement phase [416, 417, 418, 419, 420]. The emergence of Debye screening of heavy quark potential [421] giving rise to suppression of quarkonia states J/ψ also drew attention to formulation of nonrelativistic potentials [422, 423, 424, 425, 426] and lattice QCD calculations [360, 417, 418, 419, 427].

6.1 Thermal Fluctuating Glueball Masses

In this subsection, we will calculate the thermal glueball masses from the effective Lagrangian density (5.1) using similar approach as adopted in Sec. 5.2.7. However, the expectation values coming out of this process will be treated as thermal averages. We will also replace the classical tachyon field ϕ with the glueball field η defined such that $\eta = \bar{\eta} + \eta_T$. Where, $\bar{\eta}$ is the mean glueball field and η_T is the thermal fluctuating glueball field which follows the restriction $\langle \eta_T \rangle = 0$. For mathematical convenience, we

will redefine the colorless glueball field in a dimensionless form as, $\chi = \eta/\phi_0$ [428]. As a result,

$$m_\phi^{*2}\phi_0^2 = -\left\langle\frac{\partial^2\mathcal{L}}{\partial\chi^2}\right\rangle = \frac{\phi_0^2 m_\phi^2}{4}\langle F^{\mu\nu}F_{\mu\nu}\rangle + m_\phi^2\phi_0^2 - m_q m_\phi^2\phi_0^2\langle\bar{\psi}\psi\rangle; \quad (6.1)$$

where m_ϕ^{*2} , corresponds to thermal fluctuating scalar glueball mass. The mean glueball field has two possible solutions, one at $\bar{\eta} = 0$ and the other at $\bar{\eta} \simeq 1$, this can be seen by explicitly taking the thermal average of Eq.(5.2) after introducing the shift. In that case, $\bar{\eta} = 0$, corresponds to maximum condensate and $\bar{\eta} \simeq 1$ corresponds to the minimum condensate.

On the bases of dimensional analysis, it should be noted that $G(\eta)$ is dimensionless by virtue of the Regge slope $2\pi\alpha'$ which has the dimension of length square. So, Eqs. (6.1) should be analysed dimensionally by coupling $G(\eta)$ with $(2\pi\alpha')^2$ which has been set to unity throughout this thesis. Now, we determine the thermal averages of the gauge fields $\langle F^{\mu\nu}F_{\mu\nu}\rangle$ and the spinor fields $\langle\bar{\psi}\psi\rangle$ in order to be able to determine the precise expression for Eq.(6.1) in the following subsections.

6.1.1 Thermal Averages

Gauge Fields

To determine the thermal average of the gauge field, we use the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2; \quad (6.2)$$

where

$$F^{\mu\nu}F_{\mu\nu} = -2A^\nu[\Box g_{\mu\nu} - \partial_\nu\partial_\mu]A^\mu \quad (6.3)$$

and

$$\partial_\mu A^\mu\partial_\nu A^\nu = -A^\mu\partial_\mu\partial_\nu A^\nu, \quad (6.4)$$

so, Eq.(6.2) becomes

$$\mathcal{L} = \frac{1}{2}A_\nu\left[\Box g^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right)\partial_\nu\partial_\mu\right]A_\mu. \quad (6.5)$$

The last term in Eq.(6.2) is the gauge fixing term with the parameter α being a family of gauge fixing conditions mostly referred to as Feynman-'t Hooft gauge. In the limit where $\alpha \rightarrow 0$, we retrieve the strong constraint $\partial_\mu A^\mu = 0$, the Lorentz gauge. To facilitate calculations, the most convenient gauge is the Feynman gauge $\alpha = 1$. Fixing the gauge

automatically destroys the gauge invariant and can render the gauge fields massive. That notwithstanding, the family of gauges give rise to

$$G_{\mu\nu}(x-y) = \left[g^{\mu\nu} + \left(1 - \frac{1}{\alpha}\right) \frac{\partial^\mu \partial^\nu}{\partial^2} \right] G(x-y); \quad (6.6)$$

here, $G_{\mu\nu}(x-y)$ is a propagator of the gauge group and $G(x-y)$ is the propagator for the scalar field. For arbitrary α , $G_{\mu\nu}(x-y)$ is a solution of the Green's function equation

$$\left[g^{\mu\nu} \partial^2 - \left(1 - \frac{1}{\alpha}\right) \partial^\mu \partial^\nu \right] G_{\nu\lambda}(x-y) = g_\lambda^\mu \delta^4(x-y). \quad (6.7)$$

Hence, in the Feynman gauge, the above equation can be rewritten as

$$\partial^2 G^{\mu\nu}(x-y) = g^{\mu\nu} \delta^4(x-y), \quad (6.8)$$

and Eq.(6.6) becomes

$$G^{\mu\nu}(x-y) = g^{\mu\nu} G(x-y), \quad (6.9)$$

thus,

$$\partial^2 G(x-y) = \delta^4(x-y), \quad (6.10)$$

accordingly, $G(x-y)$ is the Green's function of the propagator. Additionally, we can express a corresponding Feynman propagator in the form of Fourier expansion as

$$G_F^{\mu\nu}(x-y) = -i \langle 0 | T A^\mu(x) A^\nu(y) | 0 \rangle = - \int \frac{d^4 k}{(2\pi)^4} \frac{g^{\mu\nu}}{k^2 + i\varepsilon} e^{-ik(x-y)}; \quad (6.11)$$

where T is the time ordering operator and ε is an infinitesimal positive constant which regulates the integral at $k^2 = 0$ [124, 429]. Now, supposing that $x = y$, we can drop the time ordering operator and use the expression in Eq.(6.9) so,

$$\begin{aligned} G_F(0) &= -i \langle 0 | A_\mu A^\mu | 0 \rangle = -\nu \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\varepsilon} \rightarrow \\ \langle 0 | A_\mu A^\mu | 0 \rangle &= -\nu \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \int \frac{dk_0}{(2\pi)} \frac{i}{k^2 + i\varepsilon}; \end{aligned} \quad (6.12)$$

where ν is the degeneracy. By introducing the gauge fixing term into Eq.(6.3) and selecting Feynman gauge $\alpha = 1$, we get

$$F^{\mu\nu} F_{\mu\nu} = -2A^\mu \square A_\mu, \quad (6.13)$$

so

$$\langle 0 | F^{\mu\nu} F_{\mu\nu} | 0 \rangle \simeq \langle 0 | A_\mu (-\square) A^\mu | 0 \rangle = -\nu \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \int \frac{dk_0}{(2\pi)} \frac{i k^2}{k^2 + i\varepsilon}. \quad (6.14)$$

Performing Wick rotation such that $k_0 \rightarrow -ik_0$, so $k^2 = (-ik_0)^2 - \mathbf{k}^2$, since $k^2 = 0$ it implies that $k_0 = |\mathbf{k}|$, then we can substitute $k^2 = -2\mathbf{k}^2$. Consequently,

$$\begin{aligned} \int \frac{dk_0}{2\pi} \frac{i k^2}{k^2 + i\varepsilon} &= - \int \frac{d\mathbf{k}}{2|\mathbf{k}|^2} (-2|\mathbf{k}|^2) \\ &= |\mathbf{k}|. \end{aligned} \quad (6.15)$$

The energy of the photon can be written as $E_A = |\mathbf{k}|$, therefore, we can express Eq.(6.12) as

$$\langle 0 | F^{\mu\nu} F_{\mu\nu} | 0 \rangle = -\nu \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{|\mathbf{k}|^2}{2|\mathbf{k}|}. \quad (6.16)$$

With the exception of the degeneracy and the negative sign, this equation looks the same as Eq.(4.26). So, following similar discussions contained in Sec. 4.3 i.e. $A_\mu = A_\mu^- + A_\mu^+$ and generalizing the vacuum expectation value, we can express

$$\langle F^{\mu\nu} F_{\mu\nu} \rangle = - \langle A^\mu \square A_\mu \rangle = - \frac{\nu}{2\pi^2} \int \frac{|\mathbf{k}|^4 d\mathbf{k}}{E_A} n_B(E_A). \quad (6.17)$$

Consequently, in a more general form, we can write

$$\langle F^{\mu\nu} F_{\mu\nu} \rangle = - \frac{\nu}{2\pi^2} \int \frac{k^4 dk}{E_A} n_B(E_A). \quad (6.18)$$

Accordingly, in the high temperature limit,

$$\begin{aligned} \langle F^{\mu\nu} F_{\mu\nu} \rangle &= - \frac{\nu}{2\pi^2} \int_0^\infty dk \frac{k^4}{E_A} \frac{1}{e^{\beta E_A} - 1} \\ &= - \frac{T^4 \nu}{2\pi^2} \int_0^\infty \frac{x^3 dx}{e^x - 1} \\ &= - \frac{\nu \pi^2 T^4}{30}, \end{aligned} \quad (6.19)$$

we parametrized $x = E/T$.

Dirac Fields

The thermal expectation of the spinors can be calculated from the Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_q)\psi, \quad (6.20)$$

with Feynman propagator

$$S_F = \frac{i}{\not{p} - m_q + i\varepsilon}. \quad (6.21)$$

The corresponding 4-momentum integral can be expressed as

$$S_F(x - y) = \langle 0 | T \{ \psi(x) \bar{\psi}(y) \} | 0 \rangle = i\nu_q \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-y)} \frac{\gamma \cdot p + m_q}{p^2 - m_q^2 + i\varepsilon}; \quad (6.22)$$

where ν_q is the degeneracy of the quarks and $S_F(x - y)$ satisfies $(i\partial_x - m_q)S_F(x - y) = i\delta^4(x - y)$ hence, $S_F(x - y)$ is the Green's function of the Dirac's operator. Thus, we can set $x = y$ and proceed similar to the procedure above and the one adopted in Sec. 4.3. The integral in Eq.(6.22) can be divided into odd and even integrals, ignoring the odd part, we can proceed as

$$\begin{aligned} \langle \psi \bar{\psi} \rangle &= \nu_q m_q \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m_q^2 + i\varepsilon} \\ &= -\nu_q m_q \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \int \frac{dp_0}{4\pi(\mathbf{p}^2 + m_q^2)} \\ &= \nu_q m_q \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2\sqrt{\mathbf{p}^2 + m_q^2}}. \end{aligned} \quad (6.23)$$

Now, we expand the Dirac fields in terms of plane waves in order to quantize it,

$$\begin{aligned} \psi(x) &= \psi^+(x) + \psi^-(x) \\ &= \sum_{r, \mathbf{p}} \left(\frac{m_q}{VE_{\mathbf{p}}} \right)^{1/2} [c_r(\mathbf{p})u_r(\mathbf{p})e^{-ipx} + d_r^\dagger(\mathbf{p})v_r(\mathbf{p})e^{ipx}], \end{aligned} \quad (6.24)$$

and the conjugate field $\bar{\psi} = \psi^\dagger \gamma^0$ can also be expanded as

$$\begin{aligned} \bar{\psi} &= \bar{\psi}^+(x) + \bar{\psi}^-(x) \\ &= \sum_{r, \mathbf{p}} \left(\frac{m_q}{VE_{\mathbf{p}}} \right)^{1/2} [d_r(\mathbf{p})\bar{v}_r(\mathbf{p})e^{-ipx} + c_r^\dagger(\mathbf{p})\bar{u}_r(\mathbf{p})e^{ipx}], \end{aligned} \quad (6.25)$$

and $\bar{u}_r = u_r^\dagger \gamma^0$, and so on. Also, V is the volume, u_r and v_r are specific spin states labelled as $r = 1, 2$ and $(m_q/VE_{\mathbf{p}})$ is the expansion coefficient used in the expressions with $c = \hbar = 1$. These equations obey the Pauli principle and Fermi-Dirac statistics and the expansion coefficients follow the *anticommutation relation*

$$\{c_r(\mathbf{p}), c_s^\dagger(\mathbf{p}')\} = \{d_r(\mathbf{p}), d_s^\dagger(\mathbf{p}')\} = \delta_{rs} \delta_{\mathbf{p}\mathbf{p}'}, \quad (6.26)$$

all other *anticommutation relations* vanishes. Additionally, they satisfy the orthonormal relations

$$u_r^\dagger(\mathbf{p})u_s(\mathbf{p}) = v_r^\dagger(\mathbf{p})v_s(\mathbf{p}) = \frac{E_{\mathbf{p}}}{m_q} \delta_{rs} \quad \text{and} \quad u_r^\dagger(\mathbf{p})v_s(-\mathbf{p}) = 0, \quad (6.27)$$

normalized at

$$u_r^\dagger(\mathbf{p})u_r(\mathbf{p}) = v_r^\dagger(\mathbf{p})v_r(\mathbf{p}) = \frac{E_{\mathbf{p}}}{m_q}. \quad (6.28)$$

We can also define operators

$$N_r(\mathbf{p}) = c_r^\dagger(\mathbf{p})c_r(\mathbf{p}), \quad \bar{N}_r(\mathbf{p}) = d_r^\dagger(\mathbf{p})d_r(\mathbf{p}); \quad (6.29)$$

where c_r , $c_r^\dagger$, N_r and d_r , $d_r^\dagger$, $\bar{N}_r$ are the annihilation, creation and number operators for two different particles, both fermions. However, the difference between the particle and an antiparticle density is a conserve quantity expressed as $n_b(\mathbf{p}) = N_r(\mathbf{p}) - \bar{N}_r(\mathbf{p})$. Where, $n_b(\mathbf{p})$ is the baryon number density of quark or diquark degrees of freedom [430].

Consequently, we can express

$$\begin{aligned} \bar{\psi}\psi &= \sum_{r,\mathbf{p}} \frac{1}{V} \left(d_r(\mathbf{p})d_r^\dagger(\mathbf{p}) + c_r^\dagger(\mathbf{p})c_r(\mathbf{p}) \right) \\ &= \sum_{r,\mathbf{p}} \frac{1}{V} \left(1 + N_r(\mathbf{p}) - \bar{N}_r(\mathbf{p}) \right) \\ &= \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{m_q}{2E_{\mathbf{p}}} (1 + n_b); \end{aligned} \quad (6.30)$$

in the second step we have used the definitions for the number density operator and in the last step we used the argument that, since r and $\mathbf{p}$ are continuous variables we can write $\sum_{r,\mathbf{p}}$ in terms of integral as

$$\sum_{r,\mathbf{p}} \frac{1}{V} \rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{m_q}{2E_{\mathbf{p}}}. \quad (6.31)$$

Here, there is liberty of choice, however, we used a definition suitable for our course guided by the expression in Eq.(6.23). Now, for Fermi-Dirac condensates, we can redefine the baryon number density in terms of the Fermi-Dirac distribution function as

$$n_b(\mathbf{p}) \equiv n_F(E_{\mathbf{p}}) = \frac{1}{e^{E_{\mathbf{p}}\beta} + 1}. \quad (6.32)$$

Accordingly, the thermal average Eq.(6.23) after subtracting the fundamental propagator becomes

$$\langle \bar{\psi}\psi \rangle = \frac{\nu_q}{4\pi^2} \int_0^\infty \frac{m_q p^2 dp}{E_{\mathbf{p}}} n_F(E_{\mathbf{p}}), \quad (6.33)$$

and $E_{\mathbf{p}} = \sqrt{m_q^2 + \mathbf{p}^2}$ is the single particle energy. In the high energy limit

$$\begin{aligned} \langle \bar{\psi}\psi \rangle &= \frac{m_q \nu_q}{4\pi^2} \int_0^\infty dp \frac{p^2}{E_{\mathbf{p}}} n_F(E_{\mathbf{p}}) \\ &= \frac{\nu_q m_q T^2}{4\pi^2} \int_0^\infty \frac{x dx}{e^x + 1} \\ &= \frac{\nu_q m_q T^2}{48}. \end{aligned} \quad (6.34)$$

6.2 Thermal Fluctuating Confining Potentials

6.2.1 Net Potential at Finite Temperature

We can also calculate the QCD vacuum from the classical tachyon potential (4.8) using the expression

$$\theta_\mu^\mu = 4V(\phi) - \phi \frac{dV(\phi)}{d\phi}, \quad (6.35)$$

gives rise to the relationship $\theta_\mu^\mu \approx -\alpha^2 a^2 \rho \phi^2 = -(m_\phi^2/2)\phi^2$, which is related to the Bag constant [428] as

$$-B_0 = -\frac{m_\phi^2 \phi_0^2}{2}. \quad (6.36)$$

Now, subtracting the fundamental gluon and quark condensations from Eqs.(6.19) and (6.34), and introducing critical temperatures yield,

$$\langle F^{\mu\nu} F_{\mu\nu} \rangle = 4 \langle F^{\mu\nu} F_{\mu\nu} \rangle_0 \left[1 - \frac{T^4}{T_{cA}^4} \right], \quad (6.37)$$

and

$$\langle \bar{\psi}\psi \rangle = -\langle \bar{\psi}\psi \rangle_0 \left[1 - \frac{T^2}{T_{c\psi}^2} \right], \quad (6.38)$$

corresponding to critical temperatures

$$T_{cA} = \left(\frac{15}{2\nu\pi^2} \langle F^{\mu\nu} F_{\mu\nu} \rangle_0 \right)^{1/4} \quad \text{and} \quad T_{c\psi} = \left(\frac{48}{\nu_q m_q} \langle \bar{\psi}\psi \rangle_0 \right)^{1/2}, \quad (6.39)$$

respectively. In the absence of degeneracies, the critical temperatures increase infinitely. Substituting Eqs.(6.36) and (6.38) into (6.1) yields,

$$m_\phi^{*2} \phi_0^2 = \phi_0^2 m_\phi^2 \langle F^{\mu\nu} F_{\mu\nu} \rangle_0 \left[1 - \frac{T^4}{T_{cA}^4} \right] + m_q (\phi_0^2 m_\phi^2) \langle \bar{\psi}\psi \rangle_0 \left[1 - \frac{T^2}{T_{c\psi}^2} \right]; \quad (6.40)$$

where the second term to the right of Eq.(6.1) has been absorbed into the definition of the fundamental condensates. Introducing the Regge slope, we can define a dimensionless quantity $\phi_0^2 m_\phi^2 (2\pi\alpha')^2 = 2$. Noticing that $\langle F^{\mu\nu} F_{\mu\nu} \rangle_0 \approx B_0$, we can relate

$$\frac{\langle F^{\mu\nu} F_{\mu\nu} \rangle_0}{\phi_0^2} \approx \frac{m_\phi^2}{2} \quad \text{and} \quad \frac{\langle \bar{\psi}\psi \rangle_0}{\phi_0^2} \approx \frac{m_q}{2}. \quad (6.41)$$

Equation (6.40) can be reduced to

$$m_\phi^{*2} = m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right] + m_q^2 \left[1 - \frac{T^2}{T_{c\psi}^2} \right], \quad (6.42)$$

at $T = 0$ we recover Eq.(5.20). The fundamental fermion propagator can be calculated from Eq.(5.12) with the expression $\bar{\psi}\psi \simeq q\delta(r)$ using the definition of the delta function in Eq.(5.10) leads to

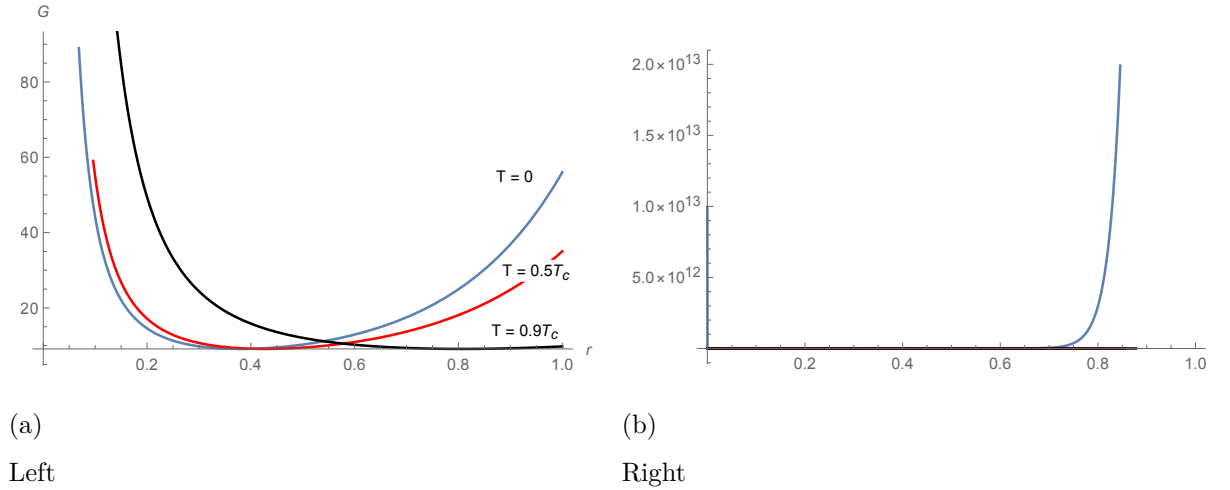
$$\begin{aligned}\bar{\psi}\psi &= \frac{3}{4\pi} \left(\frac{4\pi m_q}{3m_\phi^2(2\pi\alpha')^2} \right) \\ &= \frac{m_q}{m_\phi^2(2\pi\alpha')^2} \rightarrow \\ \frac{\langle \bar{\psi}\psi \rangle_0}{\phi_0^2} &\simeq \frac{m_q}{2}.\end{aligned}\tag{6.43}$$

In the last step we have set the dimensionless quantity $\phi_0^2 m_\phi^2 (2\pi\alpha')^2 \approx 2$ and $q = 1$.

The thermal fluctuating glueball potential becomes

$$\begin{aligned}G(r, T) &= \left(\frac{\cosh[\sqrt{2}m_\phi^* r]}{m_\phi^* r} \right)^2 \\ &= \left(\frac{\cosh \left[\sqrt{2 \left(m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right] + m_q^2 \left[1 - \frac{T^2}{T_{c\psi}^2} \right] \right)} r \right]}{\sqrt{m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right] + m_q^2 \left[1 - \frac{T^2}{T_{c\psi}^2} \right]} r} \right)^2.\end{aligned}\tag{6.44}$$

Figure 6.1: The glueball potential, $G(r, T)$, against r, T for $m_q = 0.1$ (left) and $m_q \rightarrow \infty$ (right).



The condensate increases with increasing depth of the curves. Hence, the condensate decreases from $T = 0$ (blue), $T = 0.5T_c$ (red) and $T = 0.9T_c$ (Black).

Also, the thermal fluctuating confining potential in the IR regime becomes

$$V_c(r, T) = \frac{m_\phi^* \tanh[\sqrt{2}m_\phi^* r]}{\sqrt{2}},\tag{6.45}$$

with string tension

$$\sigma_L(T, m_q) = m_\phi^{*2}. \quad (6.46)$$

At $T = 0$ we retrieve Eqs.(5.19) and (5.20). This equation consists of two different critical temperatures one representing the gluons, T_{cA} , and the other for the fermions, $T_{c\psi}$. Since there are no evidence that both critical temperatures are equal, we need to calculate a critical temperature at which the sting tension vanishes, so we set $\sigma_L = 0$, $T_{cA} = T_{c\psi} = 1$ and $m_\phi^2 = \sigma_L - m_q^2$ to facilitate the computations. Thus,

$$\begin{aligned} m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right] + m_q^2 \left[1 - \frac{T^2}{T_{c\psi}^2} \right] &= 0 \\ \sigma_L - (\sigma_L - m_q^2)T^4 - m_q^2 T^2 &= 0, \end{aligned} \quad (6.47)$$

solving this equation for $\sigma_L \sim 1 \text{ GeV/fm}$ leads to a critical temperature

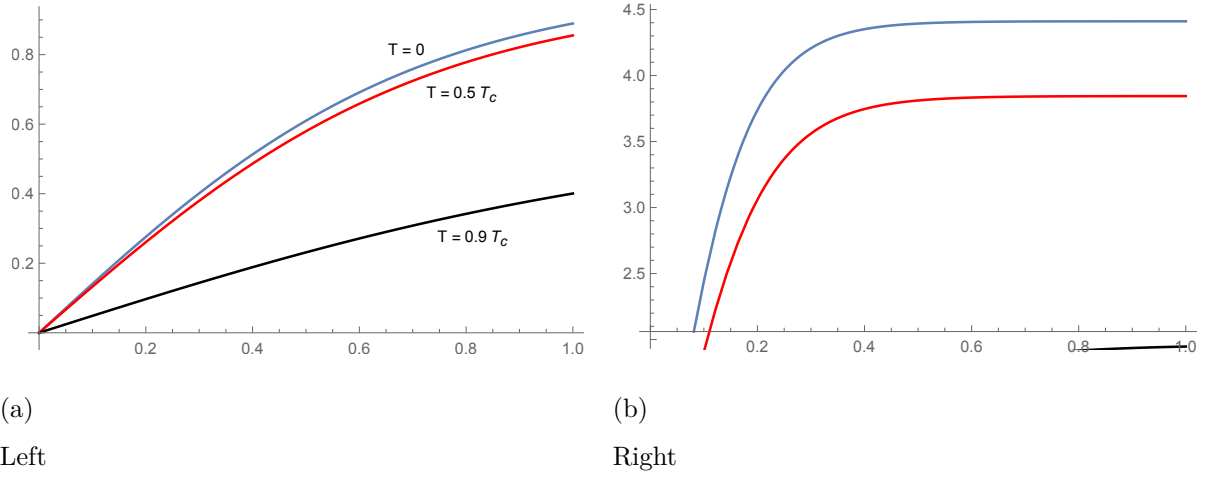
$$T_{c1} = \frac{1}{\sqrt{-1 + m_q^2}}, \quad (6.48)$$

thus,

$$\sigma_L(T) = m_\phi^2 \left[1 - \left(\frac{1}{\sqrt{-1 + m_q^2}} \right)^4 \frac{T^4}{T_{c1}^4} \right] + m_\phi^2 \left[1 - \left(\frac{1}{\sqrt{-1 + m_q^2}} \right)^2 \frac{T^2}{T_{c1}^2} \right]. \quad (6.49)$$

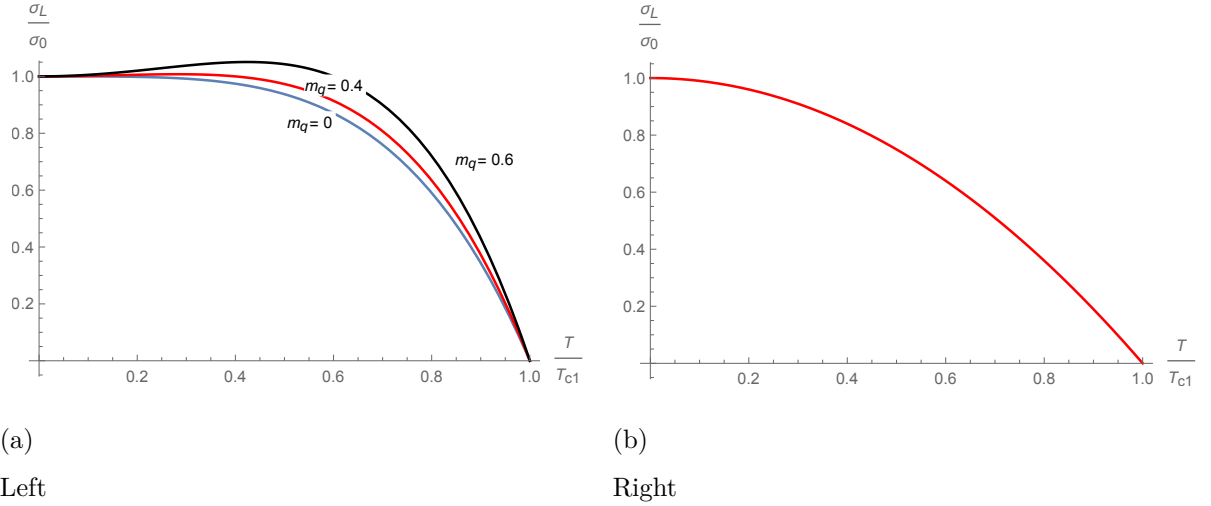
To facilitate the analyses in parts, we use $T_c = T_{cA} = T_{c\psi}$ in plotting the graphs for the potentials in this regime and in the UV regime. However, there is no evidence that both temperatures can be the same. On the contrary, it has been projected that $T_{c\psi}$ is rather greater than T_{cA} [431] indicating that, $T_{c\psi} \sim 270 \text{ MeV}$ and $T_{cA} \sim 170 \text{ MeV}$ or at least a difference of about 3 % [434].

Figure 6.2: Confining potential, $V_c(r, T)$, against (r, T) for $m_q = 0.1$ (left) and $m_q \rightarrow \infty$ (right).



The gradient of the graph increases with decreasing temperature from $T = 0.9T_c$ (black), $T = 0.5T_c$ (red) to $T = 0$ (blue) [411, 71]. Within this temperature range, there is confinement and chiral symmetry breaking. In the right panel, we show how increase in mass $m_q \rightarrow \infty$ affects the behaviour of the potential at various temperatures.

Figure 6.3: The string tension, $\sigma_L(T)/\sigma_0$, against T/T_{c1} for different m_q (left) and $m_q \rightarrow \infty$ (right).



Indeed, since the string tension is normalized, an increase in m_q goes into reducing the temperature at which the string breaks. The string breaks faster for lighter particles whilst relatively heavier ones have longer life time. In all these cases, the string tension breaks at $T = T_{c1}$ and at $T = 0$ the string tension increases linearly with m_q [2]. This curve gives the behaviour of the string tension in both the chiral $m_q = 0$ and chiral effective $m_q \neq 0$ limits. These graphs are the same as the thermal fluctuating glueball mass $m_\phi^{*2}(T)$ in the IR regime with varying m_q .

To study the potential in the UV regime, we substitute $m_\phi^2 \rightarrow -m_\phi^2$ into Eq.(6.42) giving rise to the temperature fluctuating glueball mass in the UV regime

$$m_s^{*2} = m_q^2 \left[1 - \frac{T^2}{T_{c\psi}^2} \right] - m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right], \quad (6.50)$$

so, at $T = 0$ we recover m_s^2 in Eq.(5.22). It has been acknowledged that the nonperturbative properties of the QGP is associated with changes in the properties of the scalar glueball mass and the gluon condensate [432]. This is corroborated by lattice results of pure $SU(3)_c$ theory [433], that it leads to change of sign. Therefore, the net potential in the UV regime becomes

$$V_s(r, T) = -\frac{1}{2r} + \frac{m_s^{*2}r}{3}, \quad (6.51)$$

with the string tension

$$\sigma_s(T) = \frac{1}{3} \left(m_q^2 \left[1 - \frac{T^2}{T_{c\psi}^2} \right] - m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right] \right). \quad (6.52)$$

The string becomes saturated close to $T = T_c \simeq T_{cA} \simeq T_{c\psi}$ and completely vanishes at $T = T_c$, representing thermal deconfinement. The interaction between the particles gradually weakens as the temperature increases and becomes asymptotically free at $T \geq T_c$. At $T > T_c$ the particles become asymptotically free and behave in haphazard manner [411, 435], representing the QGP phase.

Again, we can calculate the common critical temperature from which the string temperature breaks in this regime as considered for the IR regime, i.e. $\sigma_s(T) = 0$. Thus,

$$\begin{aligned} \frac{m_q^2 - m_\phi^2}{3} - \frac{m_q^2 T^2}{3} + \frac{m_\phi^2 T^4}{3} &= 0 \\ \sigma_s - \frac{m_q^2 T^2}{3} + \frac{(m_q^2 - 3\sigma_s) T^4}{3} &= 0, \end{aligned} \quad (6.53)$$

leading to a critical temperature

$$T_{c2} = \frac{\sqrt{3}}{\sqrt{-3 + m_q^2}}, \quad (6.54)$$

accordingly,

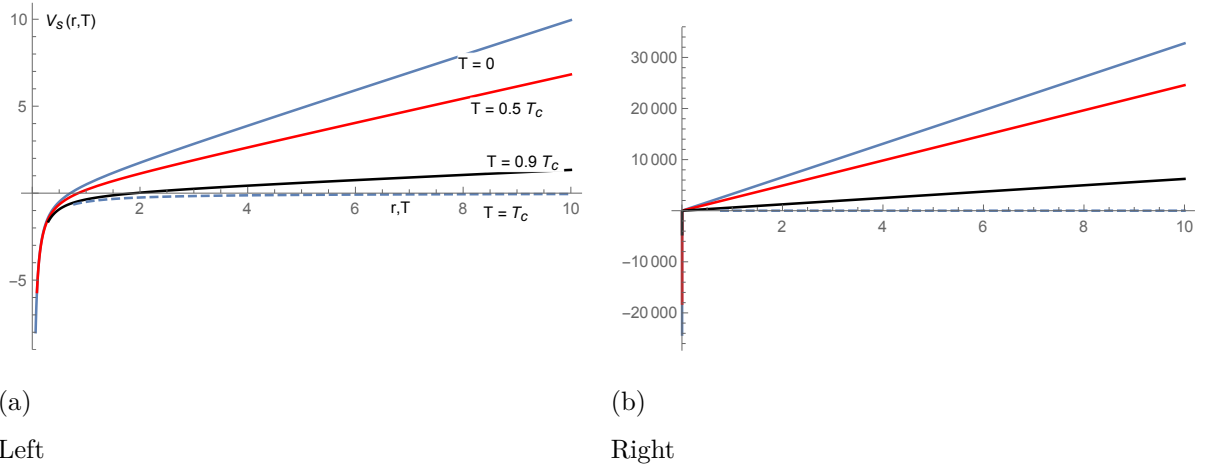
$$\sigma_s(T) = \frac{m_q^2}{3} \left[1 - \left(\frac{\sqrt{3}}{\sqrt{-3 + m_q^2}} \right)^2 \frac{T^2}{T_{c2}^2} \right] - \frac{m_\phi^2}{3} \left[1 - \left(\frac{\sqrt{3}}{\sqrt{-3 + m_q^2}} \right)^4 \frac{T^4}{T_{c2}^4} \right]. \quad (6.55)$$

The result gives the correct behaviour of the string tension as predicted by Lattice calculations [113, 366, 367]. Besides, there is some discrepancy in the order of the temperature

corrections to the string tension. In the lattice perspective, the correction is in the order of $-T^2$ but in the model framework we find a correction of $-T^2$ and $-T^4$ coexisting. Where, $-T^2$ contribution is coming from the Dirac fields and $-T^4$ is from the gauge fields. Even though $-T^4$ correction seem alien to the lattice results, some phenomenological calculations corroborate similar results [111]. Additionally, in chapter 4 we obtained a correction in the order of $-T^2$ so, we are well positioned to offer some explanations to the discrepancy by comparing the two models.

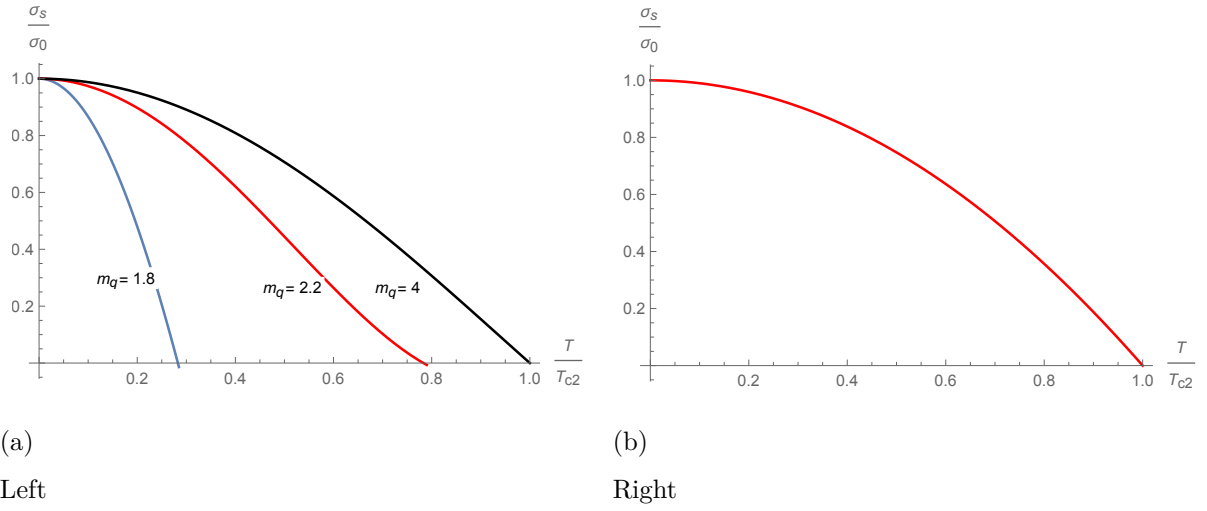
In Chap. 4 the temperature correction was introduced by the scalar field ϕ [2] which gives the expected correction in the lattice perspective. In this chapter, the temperature contribution came from both the spinor $-T^2$ and the gauge field $-T^4$. Consequently, when the temperature contribution to the gluons come from the gauge fields the correction is more likely to be in the order of $-T^4$. On the other hand, when the thermal correction to the gluon fields is introduced by the scalar field ϕ , then the correction is more likely to be in order $-T^2$.

Figure 6.4: Thermal fluctuating net potential in the UV regime, $V_s(r, T)$, against (r, T) for $m_q = 2$ (left) and $m_q \rightarrow \infty$ (right).



From the left panel the gradient of the curves decreases with increasing temperature, indicating a decrease in the binding energy that keeps the quarks together. At $T = T_c$ (dashed) we have a thermal deconfinement phase and restoration of the chiral symmetry [436, 437]. The right panel shows an increase in the strength of confinement as $m_q \rightarrow \infty$, for the same temperature variations as in the left panel.

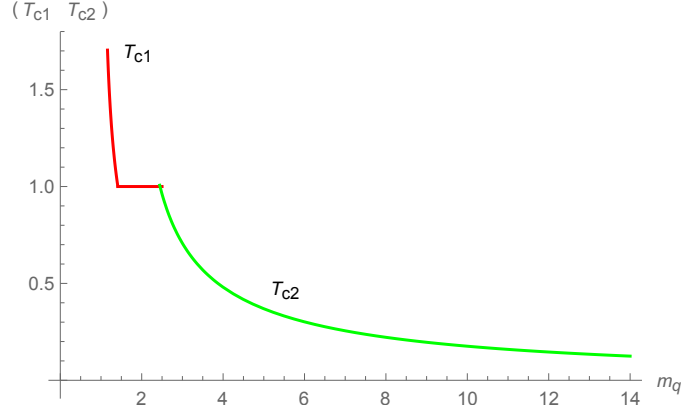
Figure 6.5: The string tension, $\sigma_s(T)/\sigma_0$, for the UV potential against T/T_{c2} for varying m_q (left) and $m_q \rightarrow \infty$ (right).



These graphs follow similar discussions as Fig. 6.3. The difference is that, the particles here are heavier and breaks the chiral symmetry explicitly as the curves clearly cut the T/T_{c2} axis at different points. Higher, m_q implies lower temperature and longer life time and the vice versa.

The critical temperatures T_{c1} and T_{c2} show the phase transition from the IR to the UV regime.

Figure 6.6: A phase diagram for T_{c1} and T_{c2} against quark mass m_q .



We show a transition from the IR regime corresponding to lighter quark masses to the UV regime corresponding to heavier quark masses. The non-physical behaviour of T_{c1} at masses $m_q \leq 1$ is the regime where confinement of light quarks are observed in the IR regime. The curve becomes constant near $T_{c1} \simeq 1$, the point where the string tension in the IR regime gets saturated and begins to degenerate. Consequently, the IR regime can take masses within $0 \leq m_q \leq 1$, beyond this threshold hadronization sets in and the string tension starts decaying. On the other hand, the UV regime takes in heavier quark masses (green curve), $m_q > \sqrt{3}$. The non-physical behaviour observed for $m_q \leq \sqrt{3}$ corresponds to the QGP regime with negative and constantly decaying σ_s . As m_q increases, T_{c2} decreases and the confinement becomes stronger.

6.2.2 Vector Potential at Finite Temperature

The vector potential has been determined in Sec. 5.2.3 so, in this section we will study its behaviour with temperature variations. We find that the vector potential is obtained at $m_q = 0$, so we can determine its thermal fluctuating glueball mass from Eq.(6.42) by setting $m_q = 0$ consequently,

$$m_{\phi A}^{*2} = m_\phi^2 \left[1 - \frac{T^4}{T_{cA}^4} \right]. \quad (6.56)$$

Substituting this equation in place of the glueball mass m_ϕ in Eq.(5.24) yields,

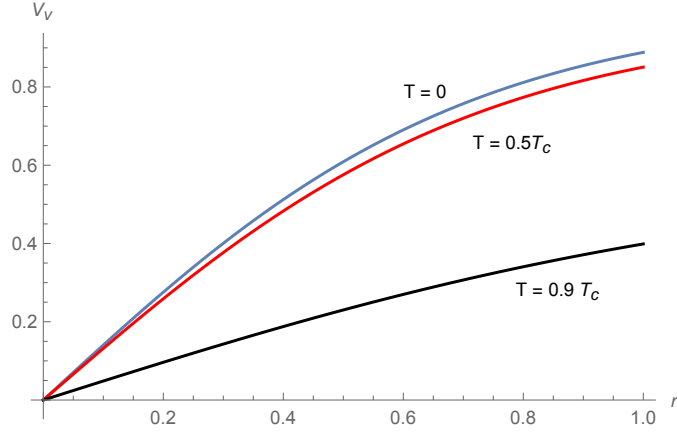
$$V_v(r, T) = \mp \frac{\lambda m_{\phi A}^* \tanh[m_{\phi A}^* r]}{2}, \quad (6.57)$$

with the string tension [438]

$$\sigma_v = \frac{m_\phi^2}{2} \left[1 - \frac{T^4}{T_{cA}^4} \right], \quad (6.58)$$

which vanishes at $T = T_{cA}$.

Figure 6.7: Thermal fluctuating vector potential.



This graph is similar to Fig. 6.2 for $m_q = 0$, its gradient increases with decreasing T . This means that the gluons remain confined even if the quark mass is 'removed' ($m_q = 0$) after confinement. This is known as *chromoelectric flux* confinement.

6.2.3 Scalar Potential at Finite Temperature

The scalar potential energy [439, 440, 441] was discussed in Sec.5.2.4 so, its thermal fluctuating equivalent will be discussed here. We substitute the glueball mass m_L^2 in Eq.(5.29) with its thermal fluctuating counterpart m_ϕ^{*2} Eq.(6.42) therefore,

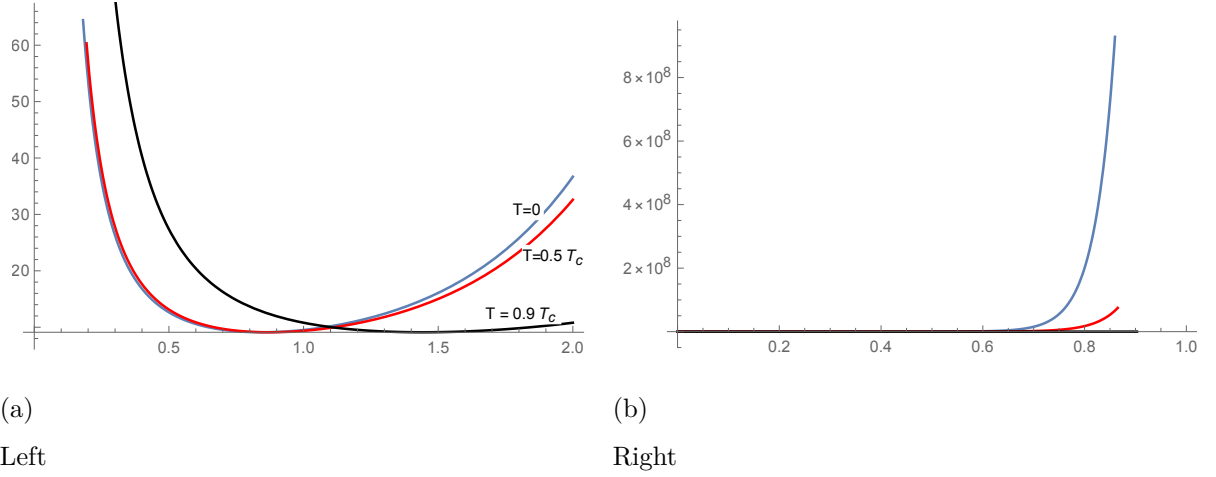
$$S(r, T) = \frac{m_q \cosh \left[\sqrt{2} m_\phi^* r \right]}{m_\phi^{*2} r^2}. \quad (6.59)$$

And the net temperature dependant confining *potential energy* becomes

$$V_{\text{net}}(r, T) = \mp \frac{q^2 m_{\phi A}^* \tanh \left[m_{\phi A}^* r \right]}{8\pi} + \frac{m_q \cosh^2 \left[\sqrt{2} m_\phi^* r \right]}{m_\phi^{*2} r^2}. \quad (6.60)$$

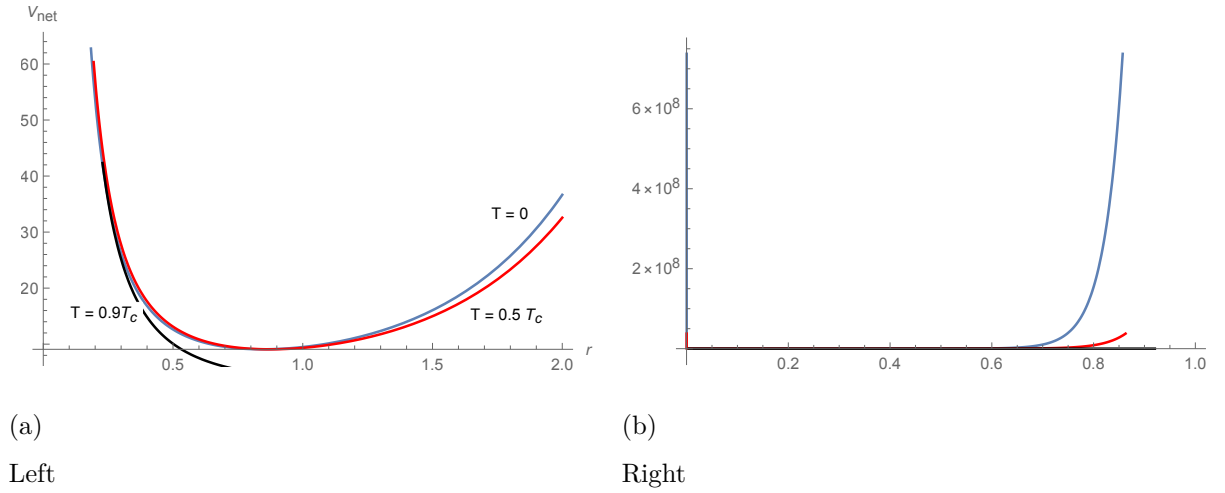
More details can be found in [3] for interested reader.

Figure 6.8: A graph of scalar potential energy, $S(r, T)$, against r, T for specific mass $m_q = 0.1$ (left) and an infinite m_q (right).



Confinement is stronger with increasing depth of the curve from $T = 0.9$ (black), $T = 0.5$ (red) to $T = 0$ (blue). Thus, the smaller the minima of the curve the more condensed the glueballs and the stronger the confinement. From both graphs, the curves decrease from left towards the minima and starts rising to the right representing screening of sea quarks [1, 352].

Figure 6.9: Thermal fluctuating net potential energy, $V_{\text{net}}(r, T)$, against r, T for $m_q = 0.1$ (left) and an infinite m_q (right).



The magnitude of the net potential decreases with increasing temperature, from $T = 0$ (blue), $T = 0.5T_c$ (red) to $T = 0.9T_c$ (black).

6.3 Hot Vacuum

The gluon condensation has been calculated in Sec. 5.3, so in this section we will study how temperature affects the condensate. In Sec. 5.3 we calculated the condensate in the IR (5.50) regime. However, we impose the restriction $\langle \eta_T \rangle = 0$ to avoid cross terms. We shall analyse this condensate in two folds; firstly, the effect of the temperature fluctuating glueball mass in the vacuum. Secondly, the effect of temperature fluctuating glueball fields in the vacuum. Accordingly, when we substitute m_L^2 with m_ϕ^{*2} in Eq.(5.50) we obtain

$$\langle G(\eta) F^{\mu\nu} F_{\mu\nu} \rangle = 8|\epsilon_v| \left\langle 1 - \frac{m_\phi^{*2} \eta^2}{4} \right\rangle. \quad (6.61)$$

So, at $T = T_c$ we observe that the glueball mass and the glueball fields dissolve into gluons to give the maximum condensate i.e.,

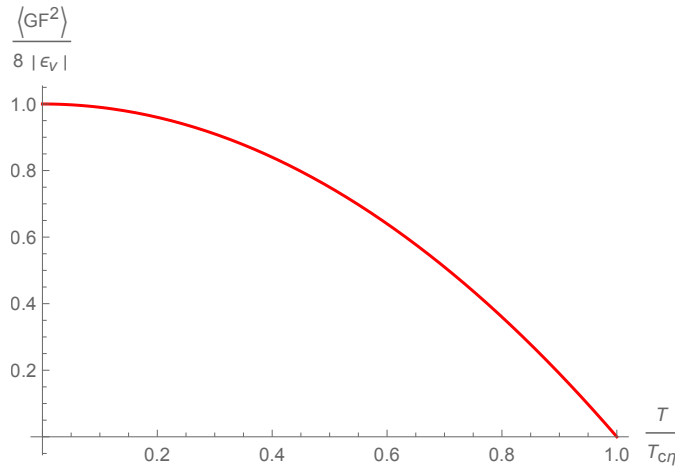
$$\langle G(\eta) F^{\mu\nu} F_{\mu\nu} \rangle = 8|\epsilon_v|. \quad (6.62)$$

Remembering that the thermal fluctuating glueball field $\langle \eta_T^2 \rangle$ is expressed in Eq.(4.55), substituting it into Eq.(5.50) leads to

$$\begin{aligned} \langle G(\eta) F^{\mu\nu} F_{\mu\nu} \rangle &= 8|\epsilon_v| \left[1 - \frac{m_L^2}{4} \left(\frac{T^2 \phi_0^2}{12 \phi_0^2} \right) \right] \\ &= 8|\epsilon_v| \left[1 - \frac{T^2}{T_{c\eta}^2} \right] \quad \text{where} \quad T_{c\eta} = \frac{4\sqrt{3}}{m_L}. \end{aligned} \quad (6.63)$$

In this expression, the condensate vanishes at $T = T_{c\eta}$. One can refer to Sec. 1.1.3 for more detailed explanations and the cosmological implications.

Figure 6.10: A graph of gluon condensate $\langle G(\eta) F^{\mu\nu} F_{\mu\nu} \rangle / (8|\epsilon_v|)$ against $T/T_{c\eta}$.



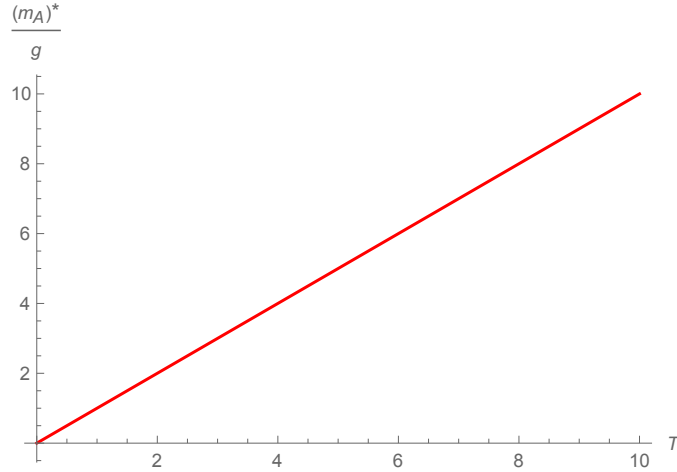
The condensate reduces sharply with increasing temperature until it vanishes at $T = T_{c\eta}$.

Also, the thermal fluctuating gluon mass can be determined from Eq.(6.63) as

$$\begin{aligned} m_A^{*2} &= \frac{m_L^2}{48\phi_0^2} T^2 \\ &= g^2 T^2; \end{aligned} \tag{6.64}$$

where we have introduced the dimensionless coupling constant, $g^2 = m_L^2/48\phi_0^2$. This outcome has a similar form as the Debye mass calculated from the leading order of QCD coupling at higher temperatures in the absence of chemical potential, $m_D = gT$ [442]. Even though, this result arises from the lowest order perturbation QCD theory [443], it is said to have an IR properties as well [444]. This mass is observed at temperatures greater than the deconfinement temperature T_c , where the bound state of the heavy quarks $Q\bar{Q}$ melt into gluons [421, 442]. This phenomenon is commonly referred to as the *Debye screening mass* of color charges.

Figure 6.11: A fluctuating gluon mass m_A^*/g with temperature T .



Increase in temperature increases the screening mass.

6.4 Chiral Condensate

Chiral symmetry breaking is a nonperturbative feature of the QCD theory together with the well-known *color confinement*. The occurrence of mass splitting of chiral partner in hadron spectrum and *Goldstone bosons* due to spontaneous symmetry breaking result into *color confinement* [403, 445, 446] in QCD vacuum [331]. Quark mass breaks the chiral symmetry dynamically or explicitly. Counting on these observations and other theoretical projections, the QCD vacuum is said to contain chiral condensate proportional

to quark condensate $\langle \bar{\psi}\psi \rangle$. Besides, temperature increase reduces the strong interaction among the hadrons and their constituent quarks due to an increase in their densities with temperature, $\rho \propto T^4$. This minimizes the condensates in the vacuum until they finally vanish at some critical temperature T_{cq} where the broken down chiral symmetry is restored. At T_{cq} the hadrons go through some phase transitions to thermal deconfinement or QGP phase where the constituent quarks and gluons behave in a haphazard manner. Additionally, increasing baryon density will have similar consequence on the QCD phase transitions as the temperature increase [447, 448].

We will compute the chiral condensate from Eq.(5.1) by taking its derivative with respect to the current quark mass m_q . Some other ways of calculating this condensates are; Chiral Perturbation Theory (ChPT) [449], Lattice QCD (lQCD) [450] and Dyson-Schwinger Equation (DSE) [451, 452]. It is a general knowledge that the dominant degrees of freedom of nonperturbative QCD is hadrons [453, 454]. Specifically, pions and kaons which interact weakly with each other, and are generally referred to as free particles under Goldstone theory. These are the dominant particles in the IR regime. Consequently, the chiral condensate can be calculated from the relation

$$\langle \bar{q}q \rangle = - \left\langle \frac{\partial \mathcal{L}}{\partial m_q} \right\rangle. \quad (6.65)$$

This equation is important for studying *spontaneous chiral symmetry breaking* $S_\chi SB$ [449]. The angle bracket represents thermal average while $\langle \bar{q}q \rangle$ is the chiral condensate. Subtracting the fundamental or the vacuum condensate $\langle \bar{q}q \rangle_0$ to ensure that the maximum condensate occurs at $T = 0$ [455] yields,

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 - \left\langle \frac{\partial \mathcal{L}}{\partial m_q} \right\rangle. \quad (6.66)$$

The details of the fundamental vacuum condensate is not important for the current analyses [456, 457], because all the information needed for the analyses are contained in the medium dependant term. Using the glueball field η for the analysis as applied in the previous sections, leads to

$$\begin{aligned} \left\langle \frac{\partial \mathcal{L}}{\partial m_q} \right\rangle &= \langle G(\eta) \bar{\psi}\psi \rangle \\ &= \left\langle \frac{m_\phi^2 \eta^2}{2} \bar{\psi}\psi \right\rangle. \end{aligned} \quad (6.67)$$

Accordingly, the condensate increases with glueball condensation $G(\eta) \rightarrow 0$ [317, 1, 2] while the intensity of the condensate is determined by the propagator, $\langle \bar{\psi}\psi \rangle$. The conden-

sate vanishes at $\langle \bar{\psi}\psi \rangle = 0$ where deconfinement and QGP phase transitions are expected, $S\chi SB$ and *color confinement* are also observed at $\langle \bar{\psi}\psi \rangle \neq 0$ [458, 459]. Under the circumstance, $\langle \bar{\psi}\psi \rangle$ serves as an *order parameter* that regulates the phase transitions. Since pion degrees of freedom play an important role in understanding chiral symmetry restoration due to the quark condensate constituted by light quarks, u and d. Consequently, protons and neutrons are also constituted by the same particles, so we can express $\langle \bar{\psi}\psi \rangle$ in terms of these particles as

$$\langle \bar{\psi}\psi \rangle = \langle \bar{q}_i q_i \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle. \quad (6.68)$$

Hence, the condensate can be seen as a constituent of two flavour quarks.

Substituting the thermal averages into Eq.(6.67) yields,

$$\begin{aligned} \langle \bar{q}q \rangle &= \langle \bar{q}q \rangle_0 - \left[\frac{m_\phi^2}{2} \left(\frac{T^2}{12} \right) \left(\frac{m_q \nu_q T^2}{48} \right) \right] \\ &= \langle \bar{q}q \rangle_0 - \left[\frac{m_\phi^2 m_q \nu_q}{1152} T^4 \right] \\ &= \langle \bar{q}q \rangle_0 \left[1 - \frac{T^4}{T_{cq}^4} \right], \end{aligned} \quad (6.69)$$

with a critical temperature

$$T_{cq} = \left(\frac{576}{m_\phi^2 m_q \nu_q} \langle \bar{q}q \rangle_0 \right)^{1/4}. \quad (6.70)$$

As a result, $T_{cq} \rightarrow \infty$ as $\nu_q \rightarrow 0$, no degeneracy, and a restoration of the chiral symmetry occur at $T = T_{cq}$. Studying the behaviour of the condensate in terms of mass, we can redefine the critical temperature as

$$T_{cq} = \left(\frac{576}{m_\phi^2 \nu_q} \langle \bar{q}q \rangle_0 \right)^{1/4}, \quad (6.71)$$

leading to the condensate

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \left[1 - m_q \frac{T^4}{T_{cq}^4} \right]. \quad (6.72)$$

lQCD affirms the spontaneous breakdown of the chiral symmetry [459, 460] to the flavour group in a form

$$SU(2)_R \times SU(2)_L \rightarrow SU(2)_V, \quad (6.73)$$

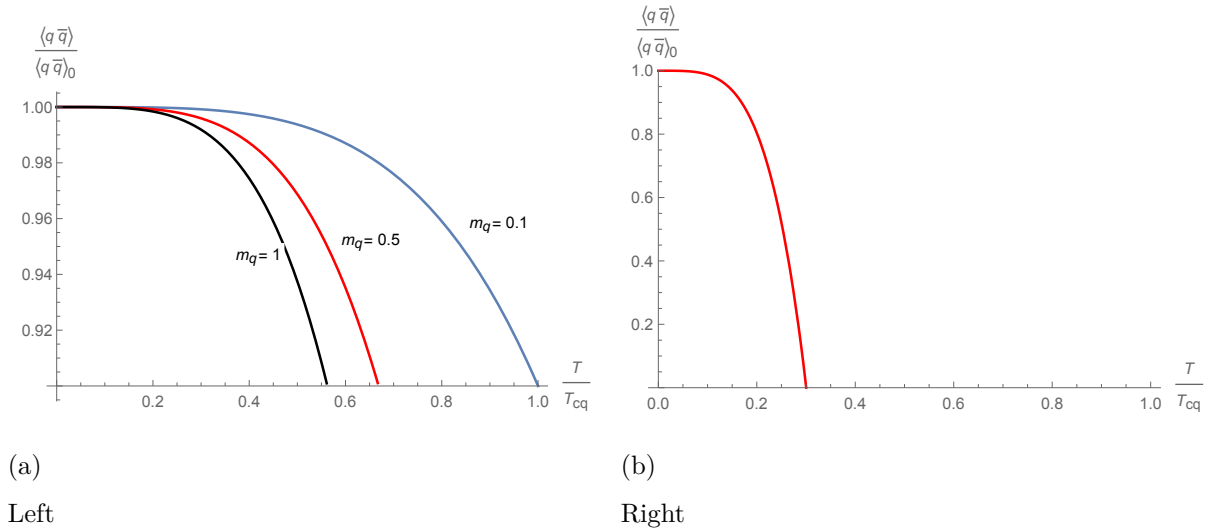
leading to the release of three Goldstone bosons ($\pi^\pm$ and π^0). In three flavour systems, the symmetry breakdown follows

$$SU(3)_R \times SU(3)_L \rightarrow SU(3)_V, \quad (6.74)$$

and the associated eight Goldstone bosons ($\pi^\pm$, π^0 , $K^\pm$, K^0 , $\bar{K}^0$ and η) [461, 123].

Subsequently, it has been observed that the high energy excited states in high energy hadron collisions result in the generation of disoriented chiral condensates which further decay into ordinary vacuum through coherent emission of low energy pions. This process is akin to Goldstone bosons observed during Higgs mechanism leaving color confining signature [462]. Although, the concept of chiral condensate was considered speculative at its inception in early 1990's [463, 464, 465, 466], it has been widely investigated both theoretically and experimentally. Despite its suitability in informing color confinement, it was also given more oxygen by the observed Centauro events in cosmic-ray [467, 468, 469], characterised by the clusters consisting pions. Due to the high energy required, chiral condensates have also been studied using high energy heavy ion collision processes, theoretically [470]. Also, the restored chiral symmetry in energetic hot QGP states make them candidates for probe. Moreover, the enormous energy and fast expanding nature of the early universe where such matter is expected to have formed, it is quenched down to low temperatures where physical objects can be observed with spontaneously broken chiral symmetries. This phenomena have attracted several attention among various groups including; cosmic-ray experiment [467, 468], nucleon-nucleon collisions at CERN [471, 472, 473], Fermi LAB [474], particularly, MiniMAX experiment [475], nucleon-nucleon collisions at CERN SPS [476, 477], and the RHIC [478, 479]. It also forms part of the heavy ion collision programme carried out with the multi purpose detector ALICE, at LHC [480, 481].

Figure 6.12: The behaviour of the chiral condensate with temperature for varying mass m_q (left) and $m_q \rightarrow \infty$ (right).



An increase in quark mass courses the condensate to reduce sharply and eventually vanish at T/T_{cq} [456, 447]. This behaviour is opposite that of the string tensions in Figs. 6.3 and 6.5 as expected in QCD lattice simulations [482]. Consequently, confinement increases with increasing quark mass while the chiral condensate decreases with increasing quark mass.

6.5 Analysis

In this chapter we investigated the phenomenon of strong interaction at finite temperature. We discussed temperature fluctuating confining potentials in two separate regimes, IR and UV regimes. Other confining properties such as string tension, glueball, glueball-meson mixing states, hot QCD vacuum, gluon mass, and the chiral condensate were also studied at finite temperature. The study throw light on the formation of QGP expected to have taken place few milliseconds after the Big Bang before particles clamped together to form protons and neutrons. Thus, this study can be extended to explain the evolution of the early universe holistically, particularly, at higher enough temperatures. Because, immensely high energy is required to dissolve hadrons into their constituent free particles, quarks and gluons, as it was formed at the time of Big Bang. Aside the high energy demand, the plasma decay very fast making it difficult for it to be created or observed in a laboratory. Hence, a phenomenological model that throws light on its characteristics by regulating temperature will be in order.

Moreover, the IR model takes in quark masses in the range of 0 to 1 GeV, while

the UV model takes in masses greater than $\sqrt{3}$, consequently, there is an intermediate mass that can be exploited between 1 and $\sqrt{3}$ GeV. This mass range, is expected to show hadronization in the IR regime and an asymptotic free behaviour in the UV regime without a stable linear confining contribution (QGP). Likewise, the energy suitable for investigating the IR properties is 1.4 GeV and below, deducing from the critical distance r_* , while in the UV regime the maximum energy for confinement is 1 GeV and below, judging from the magnitude of r_{*s} , but the process takes place at enormously high energy $r \rightarrow 0$, where Coulomb potential contribution is dominant. As a result, the model does not account for the behaviour of the particles at the intermediate energy regime, apart from knowing that the particles are 'asymptotically free' at $r \rightarrow 0$ and confined at $r \rightarrow r_{*s}$ and beyond. How the particles behave at the intermediate energy regimes is beyond the scope of this thesis. Additionally, perturbative QCD (PQCD) involves hard scattering contribution which requires higher energy and momentum transfer in order to take place. This is referred to as the 'asymptotic limit', thus, the initial and final states of the processes are clearly distinct. Therefore, it has been widely argued that the current energy regime for exploring the QCD theory describes 'sub-asymptotic' regime, calling for a revision in pure perturbation treatment of the theory. In this case, new mechanisms or models are required to investigate the dynamics of elastic scattering in the *intermediate energy* region [363, 389, 390, 391, 392]. One way to investigate this energy region ('soft' behaviour) is to use dispersion relation together with Operator Product Expansion (quark-hadron duality) which are collectively referred to as the QCD sum rule [393, 394, 395]. Thus, we can study the IR behaviour at the lower-end of the intermediate-energy region while the UV regime lies within the upper-end of the intermediate energy region to infinity. The CEBAF, Continuous Electron Beam Accelerator Facility, at Jefferson LAB was initially built to experimentally investigate these intermediate energy region before it was recently upgraded to cover the high energy regime.

Chapter 7

Glueballs and Dual Superconductivity

We have discussed the phenomenon of confinement extensively in the previous chapters, so the discussion in this chapter will be tilted more towards complex scalar fields and other associated properties that have not been highlighted yet. However, we will point out the phenomenon of confinement when necessary. Pions constitute the secondary most abundant particles generated during heavy ion collisions. They come out of hot and dense hadron gas produced in the collision processes. This is expected to be in the QGP phase or the deconfined matter state. Since pions can also be represented by complex scalar fields, the model intended for this chapter can be extended to describing it. Indeed, the photons and the dileptons that leads to the production of the pions at the QGP phase do not interact with the hadron matter through strong force, making it easy to isolate it for analysis. Pions can exist as relativistic or non-relativistic particles and can be observed through experiment [483, 484, 485, 486]. The abundance of the pions produced in the LHC and the RLHC makes it a potential candidate for the search of new physics beyond the SM.

Moreover, pion-pion annihilation is a source of dilepton production (e^+e^- and $\mu^+\mu^-$) [487, 488] as well. The study will be centred in the regime of *QGP hadronization* phase where the pions are formed by the recombination of the free quarks and gluons shortly after the Big Bang. The study will be done using complex scalar field ϕ . That notwithstanding, the glueball field η that leads to confinement comes to play after introducing a shift in the vacuum of the scalar field potential accompanied by SSB of the U(1) symmetry

imposed on the scalar sector of the theory.

7.1 The Lagrangian Density of the Model

We will adopt the Lagrangian density expressed in Eq.(3.22) for this study i.e.,

$$\mathcal{L} = \eta^{\mu\nu}(\partial_\mu\phi + iq\tilde{A}_\mu\phi)(\partial_\nu\phi^* - iq\tilde{A}_\nu\phi^*) - \frac{1}{4}G(|\phi|)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - V(|\phi|); \quad (7.1)$$

where the complex scalar field $\phi(x)$ and its conjugate $\phi^*(x)$ represent particle and its antiparticle respectively. We will study two forms of interactions, the electromagnetic interaction of the scalar fields mediated by the dual gauge field $\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}$. And the strong interaction mediated by the gauge field coupled with color dielectric function $G(|\phi|)F_{\mu\nu}F^{\mu\nu}$. The associated equations of motion are

$$D_\mu D^\mu\phi + \frac{1}{4}\frac{\partial G(|\phi|)}{\partial\phi^*}F_{\mu\nu}F^{\mu\nu} + \frac{\partial V(|\phi|)}{\partial\phi^*} = 0, \quad (7.2)$$

$$\partial_\mu\tilde{F}^{\mu\nu} = -iq[\phi^*\partial^\nu\phi - \phi\partial^\nu\phi^*] + 2q^2\phi^*\phi\tilde{A}^\nu = j_\phi^\nu, \quad (7.3)$$

and

$$\partial_\mu[G(|\phi|)F^{\mu\nu}] = 0. \quad (7.4)$$

We define a suitable scalar field potential in the form

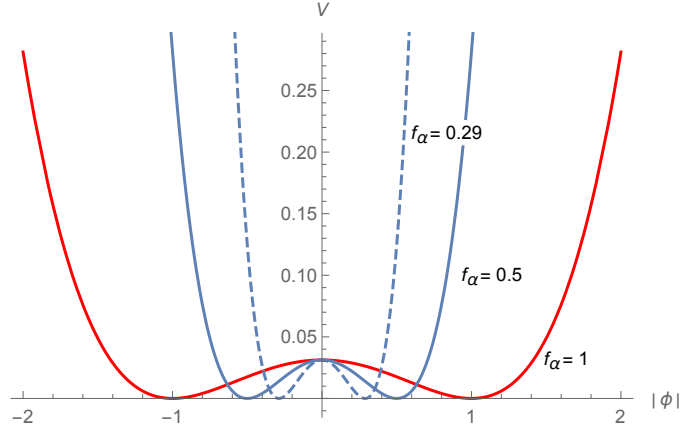
$$V(|\phi|) = \frac{\rho}{4}[\alpha^2|\phi|^2 - a^2]^2; \quad (7.5)$$

where

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}. \quad (7.6)$$

We will discuss Eq.(7.3) using perturbative approach, however, Eq.(7.2) is non-linear and hard to solve in this regime, so we will deal with it after the vacuum shift and the SSB.

Figure 7.1: The scalar field potential with varying decay constant $f_\alpha = 1/\alpha$.



The potential has a degenerate vacuum due to the presence of tachyonic modes in its spectrum.

7.1.1 Quantization of the Scalar Fields

Considering that the scalar fields are eigenstates of the momentum operator $i\partial_\mu$. And noting that the scalar fields can be represented by a plane wave $\phi = e^{-ip_\mu x^\mu}$ with eigenvalue p_μ . Accordingly, p_μ is the physical momentum of the field and the quantity $p_\mu x^\mu$ is scalar. Using Fourier transformation, we can transform the fields from spatial coordinate to a momentum space, $\phi(\mathbf{x}) \rightarrow \phi(\mathbf{p})$. Consequently, we can expand the plane wave as

$$\phi(\mathbf{x}) = \sum_{\mathbf{p}} \left(a_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} + a_{\mathbf{p}}^\dagger e^{i\mathbf{p}\cdot\mathbf{x}} \right); \quad (7.7)$$

where a and $a^\dagger$ are operators representing Fourier coefficients. Besides, the coefficients were carefully chosen to ensure that the field $\phi(\mathbf{x})$ is Hermitian at all times. Noting that $\mathbf{p}$ is a continuous variable, $\sum_{\mathbf{p}}$ can be written in terms of integral as

$$\sum_{\mathbf{p}} \rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}}. \quad (7.8)$$

This choice is quite optional but we opted for the most common definition suitable for this discussion. Also, $E_{\mathbf{p}}$, which appears in the expression is the single particle energy, $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ with momentum $\mathbf{p}$. Therefore,

$$\phi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} \left(a_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} + a_{\mathbf{p}}^\dagger e^{i\mathbf{p}\cdot\mathbf{x}} \right). \quad (7.9)$$

This gives a clear picture of a particle and its antiparticle coexisting. In a more general form

$$\phi(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} \left(a_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} + b_{\mathbf{p}}^\dagger e^{i\mathbf{p}\cdot\mathbf{x}} \right). \quad (7.10)$$

This representation is to show a clear distinction between the scalar field and its complex conjugate. Thus, $b_{\mathbf{p}}^\dagger$ and $a_{\mathbf{p}}$ represent creation and annihilation operators respectively. Accordingly,

$$\phi^*(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}} \left(a_{\mathbf{p}}^\dagger e^{i\mathbf{p}\cdot\mathbf{x}} + b_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \right). \quad (7.11)$$

This process is common in relativistic theory for massless spin-1 particles (photons and gluons) where particles and their antiparticles coexist automatically. As a result, if $b_{\mathbf{p}}$ destroys an antiparticle, $a_{\mathbf{p}}$ will destroy its corresponding particle, consequently, $b_{\mathbf{p}} \neq a_{\mathbf{p}}$.

7.1.2 Electromagnetic Interaction of the Scalar Fields

In this subsection we set out to highlight how the charged scalar particles with spin-0 interact electromagnetically. Thus, ϕ and ϕ^* represent particles with the same mass, having the same equations of motion, and have the same coupling to the electromagnetic field but opposite polarity [489]. The S-matrix of the bosonic current in Eq.(7.3) is

$$S_{int}^{(1)} = -iq \int d^4x \tilde{A}^\nu(x) \phi^* \partial^\mu \phi. \quad (7.12)$$

Transforming the fields from geometric spacetime to four-momentum space using Fourier transformation yields,

$$\begin{aligned} \tilde{A}^\nu(x) &= \int \frac{d^4k}{(2\pi)^4} e^{ikx} \tilde{A}^\nu(k), \\ \phi(x) &= \int \frac{d^4p_1}{(2\pi)^4} e^{ip_1x} \phi(p_1), \\ \phi^*(x) &= \int \frac{d^4p_2}{(2\pi)^4} e^{-ip_2x} \phi^*(p_2), \quad \text{and} \\ \partial^\mu \phi &= \int \frac{d^4p_1}{(2\pi)^4} (ip_1^\mu) e^{ip_1x} \phi(p_1). \end{aligned} \quad (7.13)$$

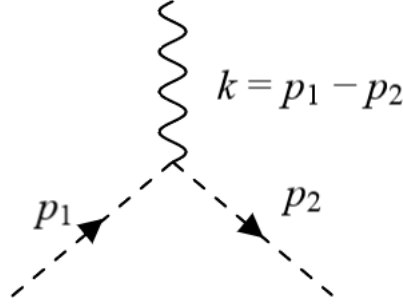
Substituting the above equations into Eq.(7.12) gives

$$\begin{aligned} S_{int}^{(1)} &= q \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{ikx} \tilde{A}^\mu(k) \int \frac{d^4p_2}{(2\pi)^4} e^{-ip_2x} \phi^*(p_2) \int \frac{d^4p_1}{(2\pi)^4} e^{ip_1x} \phi(p_1) p_1^\mu \\ &= q \int \int \int \frac{d^4k d^4p_1 d^4p_2}{(2\pi)^{12}} p_1^\mu \tilde{A}^\mu(k) \phi^*(p_2) \phi(p_1) \int d^4x e^{i(k+p_1-p_2)x}, \end{aligned} \quad (7.14)$$

as a result,

$$S_{int}^{(1)} = \frac{q}{(2\pi)^8} \int \int d^4p_1 d^4p_2 p_1^\mu \tilde{A}^\mu(p_1 - p_2) \phi^*(p_2) \phi(p_1). \quad (7.15)$$

Figure 7.2: The Feynman diagram for the interaction.

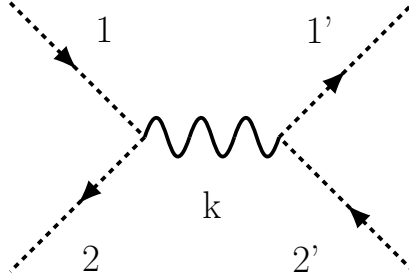


The Feynman amplitude for this interaction [429, 328] reads,

$$i\mathcal{M} = -iq(p_1^\mu + p_2^\mu). \quad (7.16)$$

Equivalent to the Bhabha scattering in QED ($e^+e^- \rightarrow e^+e^-$) [429], a process such as $\phi(p_1)\phi^*(p_2) \rightarrow \phi(p'_1)\phi^*(p'_2)$ is also possible among charged scalar fields with single photon exchange. When the electromagnetic interactions among the scalar fields are considered, ignoring all non-electromagnetic interactions, the lower order tree-level diagram is represented in Fig. 7.3 below [490].

Figure 7.3: Lower order tree-level diagram for $\phi(p_1)\phi^*(p_2) \rightarrow \gamma \rightarrow \phi(p'_1)\phi^*(p'_2)$.



Here, 1, 2, 1' and 2' correspond to momenta p_1 , p_2 , p'_1 and p'_2 respectively. Its associated Feynman amplitude in momentum space can be expressed as

$$\begin{aligned} \mathcal{M}_{fi} &= iq(p_1 + p'_1)^\mu \left[\frac{-ig_{\mu\nu}}{(p_1 - p'_1)^2} \right] (-iq)(p_2 + p'_2)^\nu \\ &= -4\pi i\alpha \frac{(p_1 + p'_1) \cdot (p_2 + p'_2)}{(p_1 - p'_1)^2}; \end{aligned} \quad (7.17)$$

where $\alpha \equiv q^2/4\pi$. At the same time, in coordinate space the interactions represented in the diagram can be expressed as

$$\text{Fig. 7.3} \equiv \int d^4x \int d^4x' j^\mu(x) D_{\mu\nu}^F(x - x') j^\nu(x'); \quad (7.18)$$

where $j^\mu(x)$ and $j^\nu(x')$ are the bosonic currents of the initial and the final processes respectively. Explicitly,

$$j^\mu(x) = -iq [\phi^*(x)\partial^\mu\phi(x) - \phi(x)\partial^\mu\phi^*(x)] \quad , \quad j^\mu(x') = -iq [\phi^*(x')\partial^\mu\phi(x') - \phi(x')\partial^\mu\phi^*(x')] , \quad (7.19)$$

and $D_{\mu\nu}^F(x - x')$ is the Feynman propagator of the photon. Using Fourier transform [328, 490]

$$\begin{aligned} j^\mu(x) &= \frac{1}{(2\pi)^4} \int d^4p j^\mu(p) e^{-ipx} , \\ j^\nu(x') &= \frac{1}{(2\pi)^4} \int d^4p' j^\nu(p') e^{-ip'x'} \quad \text{and} \\ D_{\mu\nu}^F(x - x') &= \frac{1}{(2\pi)^4} \int d^4k D_{\mu\nu}(k) e^{-ik(x-x')} , \end{aligned} \quad (7.20)$$

gives rise to the expression

$$\begin{aligned} &\frac{d^4p}{(2\pi)^{12}} \int d^4p \int d^4p' \int d^4k j^\mu(p) D_{\mu\nu}^F(k) j^\nu(p') (2\pi)^4 \delta^{(4)}(p+k) (2\pi)^4 \delta^{(4)}(k-p') \\ &= \frac{1}{(2\pi)^4} \int d^4k j^\mu(-k) D_{\mu\nu}^F(k) j^\nu(k). \end{aligned} \quad (7.21)$$

7.2 Confinement and Monopole Condensation

To investigate *color confinement*, we follow the same procedure as presented in the previous chapters, particularly Chap. 4. That notwithstanding, there is $\tilde{U}(1)$ symmetry imposed on the scalar sector and the dual gauge field $\tilde{F}^{\mu\nu}$ making the Lagrangian invariant under

$$\begin{aligned} \phi(x) &\rightarrow \phi'(x) = e^{iq\alpha(x)} \phi \quad \text{and} \\ \tilde{A}(x) &\rightarrow \tilde{A}'(x) = \tilde{A}(x) - \partial_\mu \alpha(x), \end{aligned} \quad (7.22)$$

transformations. The nonzero vacuum expectation value of the potential

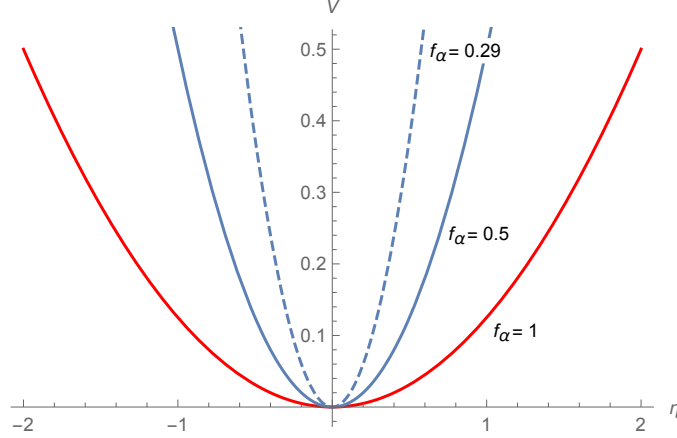
$$\langle |\phi| \rangle_0 = \pm \frac{a}{\alpha}, \quad (7.23)$$

spontaneously breakdown the $\tilde{U}(1)$ symmetry giving mass to the glueball field η . Here, we introduce a vacuum shift $\phi \rightarrow \eta + \phi_0$, leading to

$$\begin{aligned} V(\eta) &= V(\phi)|_{\langle |\phi| \rangle_0} + V'(\phi)|_{\langle |\phi| \rangle_0} \eta + \frac{1}{2} V''(\phi)|_{\langle |\phi| \rangle_0} \eta^2 \\ &= \rho \alpha^2 a^2 \eta^2; \end{aligned} \quad (7.24)$$

where η is real, representing the glueball field and $\phi_0 = \langle |\phi| \rangle_0$.

Figure 7.4: Variation of the glueball potential with the glueball field η for different f_α .



The potential has a non-degenerate and well defined vacuum, making it suitable for studying color confinement. Also, the tachyonic modes are removed giving the glueballs a real mass m_ϕ^2 .

To study color confinement among the glueballs, we shift the vacuum $\phi \rightarrow \phi_0 + \eta(r)$, and also parametrize the scalar field such that

$$\begin{aligned} \phi &= e^{i\zeta/\phi_0} \frac{(\phi_0 + \eta)}{\sqrt{2}} \\ &\approx \frac{(\phi_0 + \eta(r) + i\zeta(r))}{\sqrt{2}}. \end{aligned} \quad (7.25)$$

As a result, the Lagrangian density (7.1) takes the form

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}[\partial_\mu \eta \partial^\mu \eta - 2\rho\alpha^2 a^2 \eta^2] + \frac{1}{2}[\partial_\mu \zeta \partial^\mu \zeta - 2q\phi_0 \tilde{A}_\mu \partial^\mu \zeta + q^2 |\phi_0|^2 \tilde{A}_\mu \tilde{A}^\mu] \\ &\quad - \frac{1}{4}G(\eta)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \dots, \end{aligned} \quad (7.26)$$

where $\eta(r)$ and $\zeta(r)$ are small fluctuations about ϕ_0 [124, 125]. The glueball field play the role of massive Abelian Higgs field while $\zeta(r)$, is associated with the massless Goldstone boson field. Introducing the gauge transformation [1]

$$\tilde{A}_\mu \rightarrow \tilde{A}' = \tilde{A}_\mu - \frac{1}{q\phi_0} \partial_\mu \zeta, \quad (7.27)$$

leads to the swallowing of the Goldstone bosons by the massive gauge fields in a form

$$\begin{aligned} \frac{1}{2} \partial_\mu \zeta \partial^\mu \zeta - q\phi_0 \tilde{A}_\mu \partial^\mu \zeta + \frac{q^2 |\phi_0|^2}{2} \tilde{A}_\mu \tilde{A}^\mu &= \frac{q^2 |\phi_0|^2}{2} \left[\tilde{A}_\mu - \frac{1}{q\phi_0} \partial_\mu \zeta \right] \left[\tilde{A}^\mu - \frac{1}{q\phi_0} \partial^\mu \zeta \right] \\ &= \frac{q^2 |\phi_0|^2}{2} \tilde{A}'_\mu \tilde{A}'^\mu. \end{aligned} \quad (7.28)$$

Here, there is one Goldstone boson involved which has been swallowed by the massive gauge field, because the number of Goldstone bosons present is equivalent to the number of broken generators [322, 491].

Furthermore, the scalar field ϕ and its complex conjugate ϕ^* can be identified with the charged pions $\pi^\pm$. Because pions are bosonic and are classified as elementary particles that can interact with each other through strong force with an integer spin. They are therefore not governed by the Pauli's exclusion principle. They can exist as relativistic particles, in this case ϕ , or nonrelativistic particles, in this case the Goldstone boson ζ , signalling chiral symmetry breaking [337]. They can be produced through matter annihilation processes such as $p\bar{p}$, $N\bar{N}$, ee^+ , ect., by transforming from the baryon structure to mesons. This phenomena is of importance in low energy hadron physics investigations [338, 339]. Pions are constituted with flavour and a suitable quantum number that enable us to classify it as a bound state of quark and an antiquark. Accordingly, the valence quarks are surrounded by gluons, physically, glue-rich components mix with the pions to form an enriched spectrum of isospin-0 states and $q\bar{q}g$ hybrid states [353]. The gluons form bond states which are known in literature as glueballs, their characteristics and spectrum are well investigated under lattice QCD predictions [348, 203]. Besides, there have been challenges in affirming the existence of glueballs because of their perceived mixing with quark degrees of freedom [492]. In this thesis, we devised a theoretical strategy that enable us to isolate the glueballs for analysis. In this section, we observed that the glueballs coexist with the Goldstone bosons which are subsequently swallowed by the massive gauge fields leaving out the glueballs for analysis.

The Lagrangian can then be simplified as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - V(\eta) - \frac{1}{4}G(\eta)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{q^2|\phi_0|^2}{2}\tilde{A}_\mu\tilde{A}^\mu + \dots \quad (7.29)$$

This is similar to the Lagrangian density presented in Eq.(4.1) after introducing the vacuum shift plus the dynamics of the massive dual gauge fields. As previously discussed in Chap. 4, in the limit $\eta \rightarrow 0$, we have higher glueball condensation and stronger confinement of the glueballs. In such limit, we will be left with the Lagrangian density for the massive dual gauge field

$$\tilde{\mathcal{L}} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{q^2|\phi_0|^2}{2}\tilde{A}_\mu\tilde{A}^\mu, \quad (7.30)$$

owing to the Abelian Higgs mechanism. This represents the magnetic monopoles in the

vacuum that squeezes the uniform electric field, $\nabla \cdot \tilde{E} = 0$, at the confining phase into a flux-tube form. In general, the equations of motion for Eq.(7.29) are

$$\partial_\mu \partial^\mu \eta + \frac{\partial G(\eta)}{\partial \eta} F_{\mu\nu} F^{\mu\nu} + \frac{\partial V(\eta)}{\partial \eta} = 0, \quad (7.31)$$

$$\partial_\mu \tilde{F}^{\mu\nu} = q^2 |\phi_0|^2 \tilde{A}^\nu = j_{\phi_0}^\nu, \quad (7.32)$$

and

$$\partial_\mu [G(\eta) F^{\mu\nu}] = 0. \quad (7.33)$$

The Feynman propagator for the glueball fields is

$$D_\eta(p) = \frac{i}{p^2 - m_\phi^2 + i\varepsilon}; \quad (7.34)$$

where $m_\phi^2 = V''(\phi)|_{\phi_0} = 2\rho\alpha^2 a^2$. Hence, the glueball fields can be quantized similar to Eq.(4.28). Equations (7.31) and (7.33) leads to *color confinement* of glueballs similar to Sec. 4.2, while Eq.(7.32) will be used to study *monopole condensation* and *dual superconductivity*.

7.2.1 Confining Potentials and Glueball Mass

Expressing Eq.(7.31) in spherical coordinates and using the restricted indices defined in Sec. 4.1 together with Eq.(7.33), we obtain Eq.(4.11) which produces confinement similar to the results obtained in Sec. 4.2 and all the discussions therein. The associated effective glueball mass can be calculated from Eq.(7.29) using the expression

$$m_\eta^{*2} = - \left\langle \frac{\partial^2 \mathcal{L}}{\partial \eta^2} \right\rangle = 2\rho\alpha^2 a^2 + \frac{1}{2}\rho\alpha^2 a^2 \langle F^{\mu\nu} F_{\mu\nu} \rangle; \quad (7.35)$$

where we have substituted $\eta = \bar{\eta} + \eta_T$ with a constraint $\langle \eta_T \rangle = 0$. Using the results obtained in (6.19) for thermal average $\langle F^{\mu\nu} F_{\mu\nu} \rangle$, Eq.(7.35) reads

$$\begin{aligned} m_\eta^{*2}(T) &= 2\rho\alpha^2 a^2 \left[1 - \frac{T^4}{T_c^4} \right] \\ &= m_\phi^2 \left[1 - \frac{T^4}{T_c^4} \right], \end{aligned} \quad (7.36)$$

with

$$T_c = \left(\frac{120}{\pi^2 \nu} \right)^{1/4}. \quad (7.37)$$

So, Eq.(7.36) looks similar to the thermal fluctuating glueball mass (6.56) discussed in Sec. 6.2.2.

7.2.2 Gluon Propagator

The gluon condensation in this model framework follows the same analyses as Sec. 4.4.2. The strong interactions among the glueballs were mediated by the modified gauge field $\langle 2G(\eta)F^{\mu\nu}F_{\mu\nu} \rangle$, where the long distance dynamics of the gluons are absorbed by the color dielectric function. Thus, we can express

$$\begin{aligned}
F^{\mu\nu}F_{\mu\nu} &= -2A^\nu[\square - \partial_\mu\partial_\nu]A^\mu \\
&= -2A^\nu\left[\partial^2g^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right]A^\mu \\
&= -2A_\nu(k)\left[k^2g^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right)k^\mu k^\nu\right]A_\mu(-k) \rightarrow \\
\eta G'(\eta)F^{\mu\nu}F_{\mu\nu} &= -2A_\nu(k)\eta G'(\eta)\left[k^2g^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right)k^\mu k^\nu\right]A_\mu(-k) \\
&= A_\nu(k)\beta(q^2)\left[k^2g^{\mu\nu} - \left(1 - \frac{1}{\alpha}\right)k^\mu k^\nu\right]A_\mu(-k). \tag{7.38}
\end{aligned}$$

In the second step we have added the gauge fixing term $(2/\alpha)(\partial_\mu A^\mu)^2$, and in the third step we made use of the definition $k_\mu \rightarrow -i\partial_\mu$, $k_\nu \rightarrow i\partial_\nu$ and

$$A_\mu(k) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} A_\mu(x). \tag{7.39}$$

In the last step, the term in the square brackets is precisely the photon propagator normally associated with Abelian gauge fields but the multiplicative factor $-2\eta G'(\eta) = \beta(1/r^2) \equiv \beta(q^2)$ is defined such that the photon propagator does not decouple at higher wavelengths. At the higher wavelengths, i.e, low energies, the β -function increases in magnitude to complement the decreasing photon propagator. This influences the photons to behave like gluons.

7.3 Dual Superconductivity

We discuss briefly the phenomena of *color superconductivity* and proceed to discuss *dual superconductivity* in detail. In the model framework, the complex scalar field ϕ and its complex conjugate ϕ^* annihilate to form identical particles through decay process $\phi(p_1)\phi^*(p_2) \rightarrow \phi(p'_1)\phi^*(p'_2)$ represented in Fig 7.3. The process is mediated by a single photon exchange emitted from the dual gauge field, $\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}$. After the breakdown of the $\tilde{U}(1)$ symmetry, the dual gauge field leads to monopole condensation with the gauge field acquiring mass in the process. This process is depicted by the Lagrangian density

represented in Eq.(7.30). At the high energy regime, the multiparticle states condense through Bose-Einstein condensation [493, 494] equivalent to the isospin imbalance system, resulting into *color superconductivity* [495, 496]. At the high energy regime where the particle density is high, the particles behave like non-interacting point-like particles, asymptotically free and degenerate [497]. In this thesis, high energy or high particle density region is in reference to the region where the particles are relativistic and annihilating, whilst low energy region is in reference to the regime after SSB where the non-relativistic pions (Goldstone bosons) have been swallowed leaving glueballs for analysis. It has been established that pion condensations are constituted with nonzero isospin chemical potential μ_I , and a baryon chemical potential μ_B [498, 499, 500]. Additionally, in the two flavour quark systems, μ_I and μ_B are related to the two flavour quark chemical potential as $\mu_I = (\mu_u - \mu_d)/2$ and $\mu_B = (\mu_u + \mu_d)/2$ respectively. It has been proven [501] analytically at quark level that the critical isospin chemical potential μ_I^c [502, 503] for pion superfluidity is equivalent to the pion mass, i.e. $\mu_I^c = m_\pi$ in the vacuum.

Other characteristics of pions and their condensations have been investigated using Nambu-Jona-Lasinio (NJL) model, ladder QCD [498]. Consequently, at low particle densities, we have a confined matter and chiral symmetry breaking due to the observed Goldstone bosons and the modification of the pions into glueballs. At high particle densities the particles form a dense deconfined matter, asymptotically free and condenses through Bose-Einstein condensation, giving rise to *color superconductivity* [496, 504, 505]. The condensate has its highest occupation number at the ground state. Consequently, an increase in temperature will lead to a reduction in occupation number [506]. This phenomenon can be investigated from the propagator $\langle \phi \phi^* \rangle$. Physically, these characteristics can be observed in the core of neutron stars, other compact astrophysical objects, and heavy ion collisions in RHIC and LHC. They are suitable candidates for understanding the early universe, since they are produced out of the QGP formation.

Furthermore, we note that quark-quark pairs do not form color singlet bound states. Therefore, they form Cooper pair condensate which breaks the flavour and color symmetries to form *color superconductivity* [507]. Since, mesons are made up of different flavours and spin arrangements, different pairings are possible leading to different *color superconducting* phases. Consequently, the vacuum state constitutes a superposition of states with different quark pairings which breaks the fermion number symmetry. This

phenomenon was pioneered by Bardeen, Cooper and Schrieffer (BCS) [508]. This pairing mechanism has been proven to be more efficient in dense quark matter than superconducting metals, owing to the nature of interaction of quarks in a quark matter. Virtual fermions are created in the QCD vacuum making it color diamagnetic, screening its interior from penetration of both the electric and magnetic fields [22].

We will dedicate the rest of the section for *dual superconductivity* and discuss it within the model framework. Despite the fact that existence of monopoles have not been proven experimentally, its presence has been established theoretically by several acceptable models. The evidence of its existence are as firm as any undiscovered theoretical particles. Polyakov [509] and 't Hooft [510] suggested that monopoles are as a results of unification of fundamental forces at short distances or high energies. Dirac on the other hand, investigated monopoles in QED. Superstring and supersymmetric quantum theories provide a significant insight into the investigation of monopoles due to their relation with duality. Future discovery of monopoles will be revolutionary, because current technologies are largely grounded on electricity and magnetism.

Whilst *color superconductivity* is investigated at the deconfined matter region at low temperature, *dual superconductivity* is investigated at the confined matter region [511]. In the model framework, *dual superconductivity* will be studied at the confined glueball region, i.e. $\eta \rightarrow 0$ (a region of high glueball condensation). Dual superconductivity as a by-product of *color confinement* was pioneered by Nambu, 't Hooft and Mandelstam [165, 510, 514]. They proposed that QCD vacuum is made up of *color magnetic monopole condensation* which squeezes the *color electric flux* to form one dimensional string between quark and an antiquark pairs. This leads to the string picture of hadrons [65]. This is also associated with *dual Meissner effect*. In this vein, magnetic monopole condensation is an important feature of color confinement and *dual superconductivity*. Accordingly, we will explore the phenomenon of *dual superconductivity* in the region $r \rightarrow r_*$, where $\langle \phi \phi^* \rangle \sim \langle \phi \phi^* \rangle_0$. This will be explored by using the formalism of magnetic monopole condensation employing the well known *dual London equations* for superconductors [515]. Moreover, monopoles are involved in chiral symmetry breaking [512] and strongly coupled quark-gluon-plasma. They also play a role in decay of glasma formed at the early stages of heavy ion collisions [513].

We now focus on the Lagrangian density Eq.(7.30) and its static equations of

motion,

$$\nabla \cdot \vec{B} = \rho_m \quad \text{and} \quad -\nabla \times \vec{E} = \vec{j}_m; \quad (7.40)$$

where $\vec{E}$ and $\vec{B}$ are dual electric and magnetic fields, also, $\vec{j}_m$ and ρ_m are the magnetic current and charge densities respectively. We can also express the associated Lorentz force as

$$\vec{F} = q(\vec{B} - \vec{v} \times \vec{E}); \quad (7.41)$$

where $\vec{v}$ is the speed of the particle within the electromagnetic medium and q is the magnetic monopole charge. Accordingly, at the confining phase, we have the homogeneous equation $\nabla \cdot \vec{E} = 0$. Putting together the dual version of the Ampere's law expressed on the right side of Eq.(7.40) together with the dual London equation for persistent current generated by monopoles

$$\nabla \times \vec{j}_m = \frac{1}{\lambda^2} \vec{E}, \quad (7.42)$$

we get

$$\nabla^2 \vec{E} = \frac{1}{\lambda^2} \vec{E}. \quad (7.43)$$

Noting that $\vec{E} = \nabla \times \vec{A}$ and comparing Eqs.(7.32) and (7.42), we can deduce $\lambda = (q^2|\phi_0|^2)^{-1/2} = (q^2m_\phi^2/2\alpha^4\rho)^{-1/2}$ as the London penetration depth [516, 517]. Developing the above Laplacian leads to

$$\frac{d^2 \tilde{E}_r}{dr^2} + \frac{2}{r} \frac{d\tilde{E}_r}{dr} - \frac{\tilde{E}_r}{\lambda^2} = 0; \quad (7.44)$$

with a solution

$$\tilde{E}_r = \frac{c_1 e^{-r/\lambda}}{r}, \quad (7.45)$$

where c_1 is a positive constant. By dimensional analysis, we can fix the constant as $c_1 = \tilde{E}_0$. Hence,

$$\tilde{E}_r = \frac{\tilde{E}_0 e^{-r/\lambda}}{r}, \quad (7.46)$$

with $\tilde{E} = |\vec{E}| = \tilde{E}_r$. Consequently, the electric field is exponentially screened from the interior of the vacuum by penetration depth λ . Now using the dual Maxwell's equation on the left side of Eq.(7.40) with London's accelerating current

$$\vec{B} = -\lambda^2 \frac{\partial \vec{j}_m}{\partial t}, \quad (7.47)$$

together with the continuity equation written in dual form

$$\frac{\partial \vec{j}_m}{\partial t} + \nabla \rho_m = 0, \quad (7.48)$$

we get

$$\nabla^2 \vec{B} = \frac{\vec{B}}{\lambda^2}. \quad (7.49)$$

Similarly, its solution is given as

$$\tilde{B}_r = \frac{\tilde{B}_0 e^{-r/\lambda}}{r}. \quad (7.50)$$

Thus, the magnetic field is equally screened exponentially from the interior of the vacuum. We can simplify λ^2 obtained from the expression below Eq.(7.43) to make it suitable for comparison with the standard form, hence,

$$\lambda^2 = \frac{2\alpha^4 \rho}{q^2 m_\phi^2} = \frac{m_\phi^2}{2q^2 \rho a^4}. \quad (7.51)$$

Juxtaposing this result with the one reached in [516, 518] where

$$\lambda^2 = \frac{m}{n_s e^2}, \quad \text{with } c = \hbar = 1. \quad (7.52)$$

We can relate, $q = e$, $m = m_\phi^2/a$ and $n_s = 2\rho a^3$, where m , e , and n_s are mass, charge and number density of total conducting electrons respectively. Therefore, the electron charge is related to the monopole charge [519, 520], the conducting electron number density to conducting monopole number density and the electron mass to a monopole mass. Again, from Eqs.(7.40) and (7.42) we can express the *fluxoid quantization* relation

$$\int \vec{E} \cdot d\vec{S} - \lambda^2 \oint \vec{j}_m \cdot d\vec{l} = n\Phi_e; \quad (7.53)$$

where $n \in \mathbb{Z}$ and $\Phi_e = h/2e$ is the quantum of the electric flux [516]. Thus, the second integral is simply the enclosed monopole current if we replace $\vec{j}_m = q^2 |\phi_0|^2 \vec{A}$ and evaluate the integral. In terms of temperature, the penetration depth can be written as

$$\begin{aligned} \lambda(T) &\simeq \left(\frac{2\alpha^4 \rho}{q^2 m_\phi^2(T)} \right)^{1/2} \\ &\simeq \lambda(0) \left[1 - \frac{T^4}{T_c^4} \right]^{-1/2}. \end{aligned} \quad (7.54)$$

This is precisely the same as the temperature dependent penetration depth of a superconducting metal [518] thus, the penetration depth increases with increasing temperature and becomes infinite at $T = T_c$. At this point restoration of chiral symmetry and deconfinement coincides [436, 437]. At $T > T_c$ the electric and the magnetic field lines goes through the vacuum, this is known as *dual Meissner effect*. Moreover, the number density

of *dual superconducting monopoles* n_s can be written in terms of temperature as

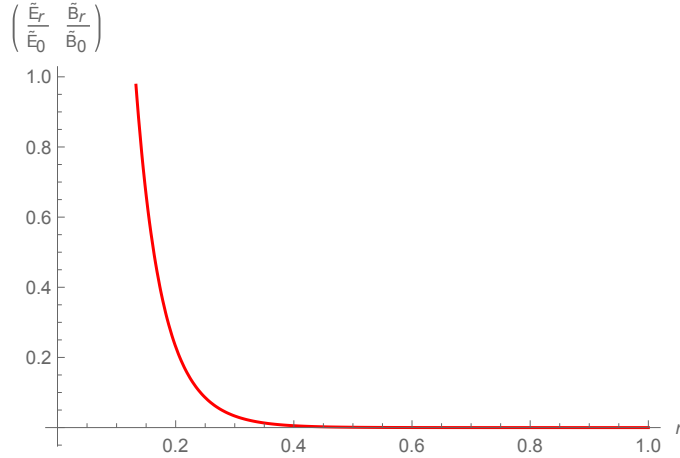
$$\begin{aligned} n_s(T) &= 2\rho a^3 \left[1 - \frac{T^4}{T_c^4} \right]^{3/2} \\ &= n \left[1 - \frac{T^4}{T_c^4} \right]^{3/2}. \end{aligned} \quad (7.55)$$

Hence, n_s decreases with temperature and vanishes at $T = T_c$. Additionally, considering two fluid systems at $T < T_c$, the normal monopole density n_n can be derived from the expression $n = n_s + n_n$ [521]. In the limit $T \rightarrow 0$ we have $n_s \rightarrow n$ and in the limit $T \rightarrow T_c$ we obtain $n_s \rightarrow 0$ therefore,

$$n_n = n \left\{ 1 - \left[1 - \frac{T^4}{T_c^4} \right]^{3/2} \right\}. \quad (7.56)$$

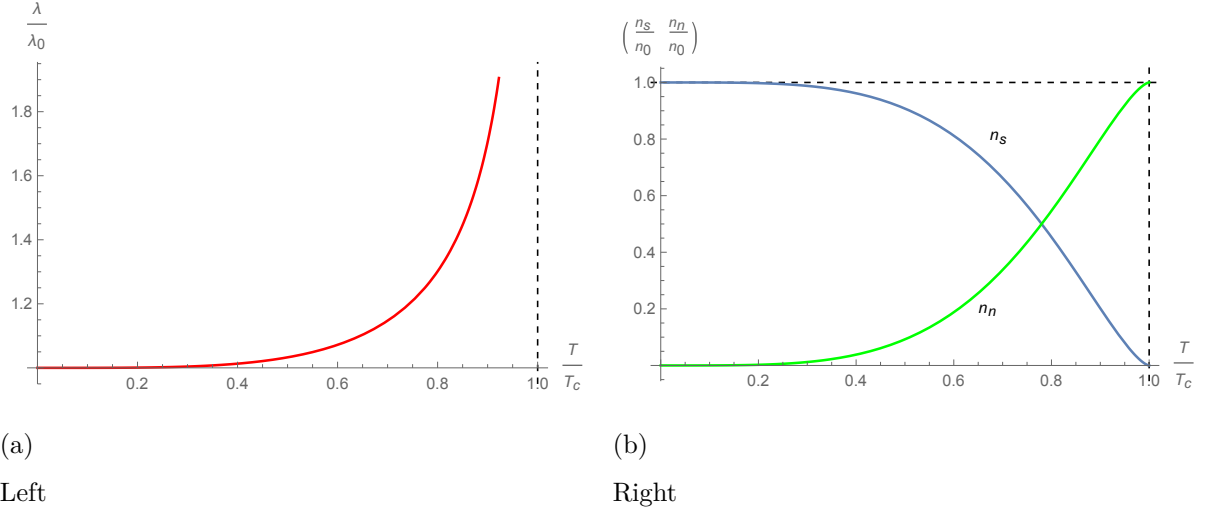
As a result, n_s at $T < T_c$, conduct with dissipationless flow, whilst n_n conduct with finite resistance at the same temperature range due to collision of the monopoles among themselves or with the lattice. Consequently, n_s decreases sharply with increasing T while n_n increases steadily with increasing T [4].

Figure 7.5: The screening effect of electric and magnetic fields.



The graph shows electric and magnetic fields screened from the interior of the vacuum. Here, we set $\lambda = 65 \text{ nm}$ in plotting the graph. Below this threshold we have a mix or intermediate states with high electric and magnetic field strengths. Above the threshold, we have a pure superconducting state because the vacuum is diamagnetic preventing the fields from penetrating.

Figure 7.6: Penetration depth λ (left) and superconducting n_s and normal n_n monopole densities (right) with temperature.



The penetration depth remains constant for a while at $\lambda = \lambda_0$ due to persistent current before it starts rising with T and becomes infinite at $T = T_c$, where the fields are expected to penetrate the vacuum. At lower temperatures where λ is constant, we have pure diamagnetic vacuum, and at relatively high temperatures we have a mix or an intermediate state. On the other hand, the superconducting monopole density n_s decreases with increasing T while the normal monopole density n_n increases with an increasing T .

7.4 Analysis

The annihilation of the scalar fields Figs. 7.3 takes place in the high particle density regime (deconfined matter) this is expected to occur at the QGP phase [522, 523]. This kind of hadronization is expected to have taken place shortly after the Big Bang when QGP formation starts cooling to the Hagedom temperature, $T \sim 170$ MeV, or high baryon density beyond $5\rho_0$. The decay process can appropriately be represented as, $\phi^*(p_1)\phi(p_2) \rightarrow \gamma \rightarrow \phi^*(p'_1)\phi(p'_2)$ [490, 524, 525, 526, 527]. This is when the free quarks and gluons start clamping together to form hadrons. This process is observed in the RHIC and the LHC during the initial stages of heavy-ion-collisions [523, 528, 529]. On the other hand, in IR regime the process is mediated by gluons which confine glueballs. The observed hadronization [530] in this regime is due to low momentum transfer among the particles. The momentum threshold for this process is determined by the QCD scale $\Lambda_{QCD} \sim 250$ MeV or particle separation greater than 1 fm. It occurs due to QCD string

decay, where hadrons are formed from quark-antiquark pairs or through gluon cascade, $g \rightarrow gq, q\bar{q}$ or gg [531, 532]. Thus, the model describes the dynamics of two forms of hadronization, one arising from the initial cooling of the QGP and the other from the low energy QCD string decay. Additionally, hadronization at the QGP phase is perturbative in nature while the string decay hadronization is nonperturbative in nature.

In the high energy regime, the particles formed are not color singlet, this can be seen by theoretically performing relativistic Fock expansion on a pion, i.e.,

$$|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}g\rangle + \cdots, \quad (7.57)$$

which covers the valence and non-valence quark components [533, 534]. Consequently, we can obtain more information on the partonic structure of hadrons stored in the pions. This forms the background for a possible transition into the deconfinement and QGP phase. QGP has enjoyed considerable attention when it was observed during ultra relativistic heavy ion collision experiment in BNL Brookhaven and CERN Geneva. This process led to the creation of matter under extreme condition of temperature and density [535]. Accordingly, pion-pion production is necessary because, pions are secondary most abundant particles produced during the collision process from hot and dense hadron gas.

The particle production process are mediated by a single photon produced from the dual gauge field $\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}$. And when the particles condense, it lead to monopole condensation and *dual superconductivity*. Besides, the gluon propagator was also generated by the modified gauge field $G(\eta)F^{\mu\nu}F_{\mu\nu}$. This leads to *color confinement* of glueballs and the breakdown of the chiral symmetry by the glueball mass. Again, this propagator leads to hadronization when the energy of the system is further reduced below some energy threshold set by QCD scale Λ_{QCD} in the IR regime. However, the chiral symmetry is restored at higher temperature regimes, $T \geq T_c$, because at this temperatures the glueball mass either dissolves or become unstable. As established with SU(2) group representation, the color charges can be screened by the gauge or quark degrees of freedom. Under this representation, one can observe quark-gluon color singlet thermionic states, quark-antiquark color singlets or gluon-gluon color singlet states.

Chapter 8

Conclusion

In this thesis, we briefly reviewed the *quark model*, where we outlined the history of quarks, their discovery, classification and their group representations to set the pace for the main study. Suitable combination of quarks leads to formation of hadrons which is also a subject for our study. We further, presented a review of chiral effective classical QCD theory to highlight the role of chiral and chiral effective limits in *color confinement* and its associated physics. The theory was used as the bases for reviewing the strong running coupling and the associated β -function. Since the models under investigation are phenomenologically inclined, we reviewed some potential models, the MIT Bag model and the Heavy Quark Effective Theory (HQET). The color dielectric medium associated with the Bag model makes it suitable for comparison with the models presented in this thesis. The models are largely based on *color dielectric function* with the same behaviour as the one that appears in the Bag model. The inclusion of the fermions to the model gives it a characteristic similar to the HQET. Holographic QCD was also given space because of its role in string theory and gauge/gravity duality which is giving QCD a new oxygen. Its main advantage is that, all the properties of strong gauge couplings can be calculated in terms of weak interacting gravity theory. Finally, we reviewed Maxwell equations in dielectric medium due to their wider applications in developing our models.

Fast forward, we developed the theories from which the studies was centred. We set off by modifying the Born and Infeld action to derive a modified Lagrangian densities which were used to phenomenologically study confinement and its associated properties. We proceeded to derive a Lagrangian for charged scalar fields which encodes both electromagnetic and strong interactions, inspired by the string representation of gauge theories

(gauge theory representation of strings). Even though, this subject has been in the scientific domain for awhile, the last word on it is yet to be spoken. In that view, it is our opinion that many inputs are needed to complement it. The theories were based on the bosonic fluctuations in an opened string with their endpoints on Dp -branes, and where fermions are required we introduce a modified SM fermions.

Furthermore, we established a relationship between glueball condensation $G(\eta)$ and the phenomena of *color confinement*. Essentially, we find that higher glueball condensation corresponds to stronger confinement and the vice versa. In a system where we introduced fermion fields with an input mass term m_q , we find that increasing mass increases glueball condensation and strengthens *color confinement*. We also established that there is more gluon condensation when the glueball fields η present in the system melts, whilst maximum glueball fields means no gluon condensation. Another important feature observed was the glueball mass (m_ϕ) and the glueball-meson mixing state (m_L and m_s) which play a central role in holding the particles together in a confined state. In a case where the system contains no mass, confinement is occasioned by the formation of bound states of gluons (glueball mass). This breaks the chiral symmetry to bring about *color confinement* as expected. Color confinement becomes easier to observe when mass is introduced into the system, because mass breaks the chiral symmetry spontaneously or dynamically depending on its magnitude. In effect, we determined that the string tension that keeps the particles in confined state is either a fraction of the pure glueball mass or the glueball-meson mixing state. Again, we calculated confining potentials in both the IR and the UV regimes. We obtain linear confining potentials in IR regime and a Cornell-like potentials in the UV regime. We also find that the strength of the Coulomb potential decreases with increasing quark mass in the system.

Other important results obtained are presented on Tab. 8.1 for easy analyses. We observed that, the glueball masses obtained in the UV regimes are higher in magnitude than the ones calculated in the IR regimes. This is quite important because higher glueball mass is required to confine particles in high energy regions. Due to the relation between glueball masses and confining strength. The same scenario is observed among the ratio of scalar glueball mass to the square root of the string tension for the same reason, because we maintained the magnitude of the string tension at $\sigma \sim 1 \text{ GeV/fm}$ throughout. This was informed by the theory where the string tension of the string connecting the branes was

set to $T_{\text{string}} = 1/(2\pi\alpha') = 1$. We also find higher screening masses in the UV regimes than the IR regimes. Accordingly, the critical distance calculated in the IR regimes represent the separation within which we are likely to observe hadronization whilst the critical distance calculated in the UV regime represents the minimum distance within which confinement begins to manifest. This is because, the particles are already in a confined state at the IR regime, while particles in the UV regime are deconfined in nature due to weak interactions. In a system without dynamic fermions, we find the critical distances to be equal in both regimes, meaning confinement in the UV regime begins when hadronization begins in the IR regime. However, in the system with fermions the distances differ in both regimes. The observed quark mass and constituent quark masses were also determined within the model framework.

Thermal fluctuating glueballs and fermion confinements were given attention in Chaps. 4, 7 and 6 respectively. These phenomena are important in theoretically creating and analysing QGP phase. Even though the study of QGP have been around for sometime, the challenges have been to create it in the laboratory for comprehensive analyses. Some of the challenges faced are:

- Tremendously high energies required to dissociate the bond between quarks to form free particles.
- The plasma decays very fast due to its short lifetime and instability.

So, apart from theoretically tracing the experimental signature of plasma formation in the mix of series of heavy ion collisions, developing some phenomenological models to throw light on this subject is in order. In the model framework, this phenomenon can be studied at extremely high temperature regimes, $T > T_c$. Where both the glueball masses, the string tensions and the QCD-like vacua appear negative and unstable. Additionally, we observed two different forms of temperature corrections to the string tensions and glueball masses in the model framework i.e., $-T^2$ and $-T^4$. The significance of these corrections and their discrepancies have been analysed in Chaps. 4, 6 and 7. The gluon condensations were also computed at finite temperature. In a case where there are thermal fluctuating glueball masses and glueball fields in the vacuum, $\langle m_\phi^2 \eta^2 F^{\mu\nu} F_{\mu\nu} \rangle$, the condensate increases with increasing temperature and at some critical temperature, the glueball masses and the glueball fields dissolve to increase the condensate. However, in the situation where

there are no glueball masses or glueball fields, the condensate decreases with increasing temperature and vanishes at the critical temperature. We also calculated Debye-like masses m_A^* , expected to have occurred at a temperatures where the quark and gluon bound states melt into gluons. In this case, the mass increases with increasing temperature.

Finally, we investigated the annihilation of pions at high energy regime and its transformation into glueballs at low energies giving an insight into confinement in Chap. 7. This was possible due to the presence of a massive gauge fields that swallowed the pions through Abelian Higgs mechanism leaving out glueballs for analysis. We also highlighted the Bose-Einstein condensation of the pions at higher pion density regime, leading to *color superconductivity*. We established monopole condensation in the QCD-like vacuum and its exhibition of *dual Meissner effect*, using London equations. We also studied the penetrations depth, the superconducting and normal monopole densities and their association with temperature. At an extreme temperature where QGP is likely to form, the penetration depth $\lambda(T)$ becomes infinite at $T = T_c$ or negative at $T > T_c$ where the field lines go through the vacuum thereby loosing its superconducting properties. The superconducting monopole density decreases with temperature whilst the normal monopole density increases with temperature. This is premised on the postulate that, a superconducting material contains both normal and superconducting currents. Proper understanding of the structure of pions and their characteristics is particularly necessary in understanding the dynamics of the early universe and the physics beyond the standard model because they emerge from the QGP phase.

Table 8.1: Table of Results Calculated for $\sigma \sim 1 \text{ GeV/fm}$.

Dimensionless ratio $m(0^{++})/\sqrt{\sigma}$		
Ratio	Value	Medium
$m_\phi/\sqrt{\sigma_L}$	1	Quenched (IR regime)
$m_\phi/\sqrt{\sigma_s}$	1.73	Quenched (UV regime)
$m_L/\sqrt{\sigma_L}$	1	Unquenched (IR)
$m_s/\sqrt{\sigma_s}$	1.73	Unquenched (UV)
Critical Distances		
Critical distance	Value (fm)	Medium
$r_* = 1/\sqrt{\sigma_L}$	1	Quenched (IR regime)
$r_{*s} = 1/\sqrt{\sigma_s}$	1	Quenched (UV regime)
$r_* = 1/\sqrt{2\sigma_L}$	0.71	Unquenched (IR regime)
$r_{*s} = 1/\sqrt{\sigma_s}$	1	Unquenched (UV regime)
Glueball Masses		
Glueball mass	Value (GeV)	Medium
m_ϕ	~ 1	Quenched (IR regime)
m_ϕ	1.73	Quenched (UV regime)
m_L	0.995	Unquenched (IR regime)
m_s	1.73	Unquenched (UV regime)
Gluon (Screening) Masses $m_A^2 = m(0^{++})^2/4$		
Gluon mass	Value (MeV)	Medium
m_A	500	Quenched (IR regime)
m_A	866	Quenched (UV regime)
m_{AL}	500	Unquenched (IR regime)
m_{AL}	866	Unquenched (UV regime)

Current Quark Masses (m_q)		
Quark mass	Regime	Compared physical particle
100 MeV	IR	strange quark
2 GeV	UV	charm quark
Constituent Quark Masses		
Quark mass $M = 2m_q$	Regime	Compared physical particle
200 MeV	IR	strange quark
4 GeV	UV	bottom quark

8.1 Projections

- The theories can eventually be applied in studying some hadronic properties.
- The Cornell-like potentials obtained from the model framework can be used to calculate the quarkonium spectroscopy.
- With the availability of computational artefacts, the theories can be discretized through path integral formalism and studied under IQCD.
- Also, the theories can be used to explore the evolution of the early universe when radiation and matter terms in the theories are integrated out to serve as temperature corrections to the scalar field ϕ . In this scenario, the temperature corrections will represent interaction of particles with ϕ (in this case, inflaton). The interaction of the scalar field with the heat reservoir will lead to restoration of broken down symmetry in the original potential at sufficiently high temperatures, $T \geq T_c$. So, one can explore the inflatory regime of the infant universe through series of phase transitions. It is worth noting that in this kind of study, one does not necessarily need to choose a dielectric function which is the same as the potential adopted for the study.

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